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PHOTOGRAPHIC FEATURE

Electro-Osmotic (EO) Consolidation with Prefabricated Vertical Drain (PVD)
by D.T. Bergado¹ and A. Patawaran²

Prefabricated Vertical Drains (PVD) have already been used and have been proven to be successful in various projects in Southeast Asia. The use of PVD coupled with electro-osmosis has been experimented in the laboratory to investigate its effectiveness. Electro-osmosis is the process wherein positively-charged free water moves from the anode to the cathode upon application of a direct current. Consolidation results when water is drained at the cathode but not replaced at the anode. Figure 1 shows the PVD cut to 20 mm width sizes and with pre-inserted copper wires to serve as electrodes. The experimental set-up is shown in Fig. 2. The apparatus consists of a modified large-scale consolidometer, made up of 10 mm thick transparent PVC sheet, 950 mm high, with an inner diameter of 450 mm placed over a steel plate. Load was applied through a loading piston with four shafts arranged in a square pattern and 200 mm center to center spacing corresponding to PVD installation. Electric current was applied by connecting the copper wires to a power supply.

Fig. 1 Modified PVD Specimens for Electro-Osmotic (EO) Consolidation

Fig. 2 Experimental Set-up of Electro-Osmotic (EO) Consolidation

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ON LARGE-SCALE LABORATORY PULLOUT TESTING

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ABSTRACT

In the construction of reinforced soil structures and geomembrane lined waste containment facilities, the soil-reinforcement interface strength is an important design parameter. Large-scale laboratory pullout testing is sometimes needed for simulating the three dimensional nature of the soil-reinforcement interaction mechanism, and to account for the progressive mobilization of pullout resistance in the case of extensible reinforcement. However, the interface strength as measured in a pullout apparatus may be dependent on the design, in particular the boundary conditions, of the pullout box. A review of major pullout apparatuses reported in the literature is presented to identify debatable issues on pullout testing. The significance of these issues was addressed by comparing pullout test results obtained with different apparatuses and simulating the influence of certain design attributes by non-linear finite analyses. A number of confusing issues such as the influence of front wall roughness, sleeve design and short specimen in a long pullout box were elucidated.

INTRODUCTION

A considerable growth in the use of various materials as soil reinforcements is a recent phenomenon in the geotechnical engineering practice. These materials could be metallic (Schlosser and Elias, 1978; Lo, 1990), bamboo (Bergado, et al., 1987), or polymeric (Holtz, 1977). Polymeric reinforcements are collectively termed as geosynthetics. In the design of reinforced soil structures and geomembrane lined waste containment facilities, the soil-reinforcement interface strength is an important parameter. The pullout test is considered to be most suited to studying the interface strength and its mobilization. Typical components of a pullout test apparatus is

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schematically presented in Fig. 1. Several researchers have designed and commissioned large-scale pullout test equipment (Bergado, et al., 1987; Palmeira and Milligan, 1989; Lo, 1990; Juran, et al., 1991; and Fannin and Raju, 1993). Large-scale laboratory testing is sometimes needed to ensure that field conditions can be reasonably simulated. The ASTM is currently developing a standard test method for large-scale laboratory pullout test. The term large-scale pullout testing is commonly restricted to pullout box with a minimum planar dimension of 0.5 m x 1 m, and hence only apparatus fulfilling this criterion are considered in this paper.

1 Base frame  
2 Flexible surcharge bag  
3 Test specimen  
4 Clamped end  
5 Embedded end  
6 Actuator/Force  
7 Front wall  
8 Sleeve (exit-slit)  
9 Soil sample  
10 Rear LVDT (optional)

Fig. 1 Pullout Apparatus

The mobilization of shear stress along the embedded length of the extensible reinforcement during pullout testing is not uniform. In this context, most geosynthetic reinforcements are extensible. The hypothesis by Juran, et al. (1988, 1989) on the development of arching during pullout implies that the normal stress acting directly on the soil-reinforcement interface may also be non-uniform. Hence, the pullout test is a model test in which the measured response may be dependent on the pullout apparatus
ON LARGE-SCALE LABORATORY PULLOUT TESTING

design, and the test results require careful interpretation. Internal measurements, in the form of strain gages or tell-tales mounted on the reinforcement or tell-tales are of great benefit to a proper understanding of the behavior in pullout testing. However, these internal measurements can only be reliably incorporated in a research laboratory and still fall short of providing a complete stress and strain field in a pullout apparatus. Therefore, considerable care has to be taken in designing the pullout apparatus so that consistent results could be obtained with minimum instrumentation.

The objectives of this paper are to synthesize relevant issues governing the design of the test apparatus, compare test results obtained from different pullout apparatuses, and attempt to explain the observations by a series of non-linear finite element analyses. Based on the comparison of test results and synthesis of non-linear finite element analyses, inferences on the design of pullout box and interpretation of internal measurements were made.

INTERPRETATION OF TEST RESULTS

In a limit equilibrium design model, the resistance against reinforcement pullout, $R_p$, is commonly calculated using the following equation (FHWA 1990, 1990a).

$$ R_p = F \alpha \sigma_{vo} p_e L_e $$

(1)

where $\sigma_{vo}$ is the average overburden stress acting at the reinforcement level, $p_e$ is the effective perimeter of the reinforcement, and $L_e$ is the reinforcement length in the anchorage zone. The parameter $(F\alpha)$, sometimes referred to as the interaction factor, relates average normal stress to average shear strength along the interface. It is a product of two parameters: $F$ as a basic interface parameter and $\alpha$ as a scale correction factor due to reinforcement extensibility. For inextensible reinforcement such as metallic strips, $\alpha = 1$ whereas $\alpha < 1$ for extensible reinforcement. In this respect, most geosynthetics are extensible. Hence large scale pullout testing is commonly interpreted with Eq. (2) below.

$$ F\alpha = \frac{P_f}{\sigma_{vo} p_e L_e} $$

(2)

where $L_e$ is the embedded length of the reinforcement in contact with surrounding soil, $\sigma_{vo}$ is the normal stress applied to the top boundary of the pullout box plus a uniform stress due to self weight of soil, and $P_f$ is the maximum (failure) value of the pullout force measured in the test. Hence $(F\alpha)$ is the normalized pullout resistance. It inherently assumed that the pullout box is of adequate length so that the scale correction factor can be considered as representative of field situation. Eq. (2) is essentially identical to Eq. (1) except the notation $P_f$ is used in lieu of $R_p$. This difference in
notation is to recognize that the conditions simulated in a pullout box may not necessarily be identical to that implied by the design equation. The term mobilized interaction factor, \((F\alpha)_{mob}\), defined in Eq. (2a) below is also used in describing the mobilization of pullout force, \(P\), with displacement.

\[
(F\alpha)_{mob} = \frac{P}{\sigma_{vo}P_eL_e}
\]  

At failure, \((F\alpha)_{mob} = (F\alpha)\). As such, Eqs. (2) or (2a) provide a common basis for normalizing \(P_f\) or \(P\), and experimental data obtained under different conditions can be conveniently compared. It also formally recognizes the need to consider scale effect. For certain geosynthetic soil combination under adequate overburden stress, the design criterion may be the rate of mobilization of pullout resistance with displacement (Bergado and Chai, 1994). Such a design consideration is beyond the scope of this paper.

**PULLOUT APPARATUS**

The attributes of a number of large-scale pullout apparatuses were listed in Table 1 in chronological order based on the date of the first refereed publication. The wide variety of pullout apparatus used gives an indication of the complexity of the design issues, which can either be of a practical or a theoretical nature.

Issues of a practical nature are the design of clamps, loading system, and instrumentation. An effective clamp is needed to ensure uniform distribution of pullout across the width of the reinforcement. Control of loading mode and rate requires a sophisticated loading system. Adequate instrumentation enables the reliable measurement of pressure, force, and displacement parameters. These practical design issues relate to inevitable testing errors which can be reduced to zero values in an idealized situation.

Theoretical design issues impinge directly on the boundary conditions of the pullout box system. Since the stress field inside a pullout box is not uniform, the boundary conditions affect the extent and nature of non-uniformity and hence the test results. This implies that the theoretical design issues may affect the test results even if a pullout test is conducted perfectly. Based on Table 1, the important theoretical design issues are:

- the method of pressure application along the top boundary
- dimensions of the pullout apparatus
Table 1 Summary of the Pullout Test Apparatus and Testing Characteristics

<table>
<thead>
<tr>
<th>Researcher(s)/ Institution</th>
<th>Dimensions (m) L x W x D</th>
<th>Soil Type and Preparation</th>
<th>Front and Side Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geosynthetic Research Institute/Drexel University, U.S.A</td>
<td>1.90 x 0.91 x 1.10</td>
<td>Concrete sand; Sand placed by manual tamping in series of 25-mm lifts</td>
<td>Metal sleeve 0.10 m long;</td>
</tr>
<tr>
<td>University of Technology, Sydney, Australia</td>
<td>1.00 x 0.60 x 0.75</td>
<td>Fly ash; compacted using jackhammer</td>
<td></td>
</tr>
<tr>
<td>(Hausmann and Clarke, 1994)</td>
<td></td>
<td>Poorly graded sand placed by compaction</td>
<td></td>
</tr>
<tr>
<td>Cullin and Berg, 1993</td>
<td>2.10 x 0.9 x 0.5</td>
<td>Uniform coarse silica sand; Air pluviation with specially designed hopper</td>
<td></td>
</tr>
<tr>
<td>The University of British Columbia, Canada</td>
<td>1.30 x 0.64 x 0.60</td>
<td>Poorly graded sand placed by compaction</td>
<td>Metal sleeve 0.125 m long;</td>
</tr>
<tr>
<td>(Funnin and Raju, 1993)</td>
<td></td>
<td>Uniform fine sand with silt traces; Placed at 95% of standard Proctor dry density and within 2% of OMC</td>
<td>sides glued with glass sheet; front and rear aluminum</td>
</tr>
<tr>
<td>Louisiana State University, U.S.A</td>
<td>1.52 x 0.61 x 0.30</td>
<td>Blasting sand, $d_{50}=0.26$ mm, varying density</td>
<td>Lubricated tapered metal sleeve on front wall;</td>
</tr>
<tr>
<td>Nitchon, U. K.</td>
<td>1.52 x 0.90 x 0.76</td>
<td>Well graded sandy gravel, $C_{	ext{w}}=28$, maximum grain size= 30 mm</td>
<td></td>
</tr>
<tr>
<td>University College, University of New South Wales, Australia</td>
<td>2.00 x 1.00 x 1.00</td>
<td>Well graded sandy gravel, $C_{	ext{w}}=28$, maximum grain size= 30 mm</td>
<td></td>
</tr>
<tr>
<td>(Lo, 1990)</td>
<td></td>
<td>fine sand to sandy gravel; placed in 100 mm thick layers; compaction with falling weight</td>
<td>lubricated front and side walls</td>
</tr>
<tr>
<td>Oxford University, U. K.</td>
<td>1.00 x 1.00 x 1.00</td>
<td>Leighton Buzzard Sand 14/25, uniform coarse; Air pluviation with specially designed hopper</td>
<td>Sides and front - Double layer of polyethylene with grease</td>
</tr>
<tr>
<td>Public Works Research Institute, Japan</td>
<td>1.20 x 0.60 x 0.60 &amp;</td>
<td>Air-dried Toyoura Sand; $d_{50}=0.16$ mm; relative density 20% and 60%</td>
<td></td>
</tr>
<tr>
<td>Kutsara et al. 1988</td>
<td>1.00 x 0.80 x 0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STS Consultants Ltd., Illinois, USA</td>
<td>1.34 x 0.70 x 0.38</td>
<td>Fontainbleau sand (SP); dense sample; Compacted using vibratory plate</td>
<td>Metal sleeve</td>
</tr>
<tr>
<td>Asian Institute of Technology, Thailand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Bergado et al, 1987)</td>
<td>1.00 x 0.80 x 0.90</td>
<td>Light brown clayey sand and weathered clay; 15 cm layer compacted to 95% of Standard Proctor at OMC</td>
<td>front - steel surface and sides - concrete</td>
</tr>
<tr>
<td>Swedish Geotechnical Institute, Sweden</td>
<td>1.90 x 0.70 x 0.70</td>
<td>Tulling sand (Stockholm) G-12 sand (Denmark); medium dense</td>
<td>side clearance of 50 mm</td>
</tr>
</tbody>
</table>
• roughness of front wall

• exit slit configuration

Pressure can be applied to the top boundary in either a prescribed uniform stress or a prescribed uniform displacement mode. The former, often referred to as a flexible top boundary, is realized by pressure application through a pressure bag or diaphragm system. The latter, often referred to as a rigid boundary, is realized by imposing uniform displacement with a very stiff loading plate. As such, the boundary pressure for a rigid boundary may not be uniform and hence $\sigma_{yo}$ of Eq. (2) and Eq. (2a) is only an average value specified to a particular apparatus. Shear stress of unknown magnitude may also be induced along the top boundary. A rigid boundary was examined by Palmeira and Milligan (1989) and was found to be problematic. Flexible boundary has been adopted in most pullout apparatuses (Lo, 1990; and Fannin and Raju, 1993). Hence the issue of pressure application along the boundary conditions will not be further investigated in this paper.

It is commonly assumed that the minimum planar dimensions of a large pullout box are 1 m length x 0.5 m wide. However, there are uncertainties about the influence of box depth. Johnston (1985) recommended the reinforcement should have a minimum distance from the top boundary to minimize "top boundary effects". For a top boundary subject to a uniform pressure, such a requirement appears to be redundant. Two box depths will be investigated in this paper.

The front boundary is usually rigid with surface characteristics varying between smooth and rough. Special glass or polished steel surface has been used to achieve a low friction boundary. Alternatively, polyethylene/rubber sheets with grease have been used to create lubricated boundary based on the principle of free ends in triaxial testing (Palmeira, 1987; and Lo, 1990). Such an arrangement, by reducing shear stresses on the front boundary, also reduces the complimentary shear stresses on the specimen close to the front boundary of the apparatus. According to the stress arching hypothesis of Jurjan, et al. (1988, 1989), a rough front wall would amplify stress arching, which in turn would reduce the pullout resistance. However, tests performed by Palmeira (1987) showed a dramatic increase in pullout resistance associated with rough front wall. This could be attributed to an increase in the normal stress on the sample caused by shear stresses developed on the front boundary during pullout. The influence of front boundary roughness will be examined in this paper.

An exit sleeve shown as Item 8 in Fig. 1 is sometimes used to separate the reinforcement from the surrounding soil along a front zone. Such an arrangement will eliminate stress transfer between reinforcement and surrounding soil along the length of the sleeve. Hence one can argue that the sleeve reduces the effect of front boundary on the test results. The exit sleeve is usually a rigid attachment (Bonczkiewicz, et al., 1988), hence referred to as a rigid sleeve. Some researchers
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(Fannin and Raju, 1993) did not use any sleeve to avoid interference to vertical displacement and stress fields. Lo (1990) debonded the front length of the reinforcement by a rubber sheet/grease arrangement. Hence the shear stress transfer for a short front zone can be eliminated without interfering the displacement field. This is referred to as a flexible sleeve. These three types of exit slit, which correspond to three different boundary conditions, will be studied in this paper.

COMPARISON OF TEST RESULTS

To examine the influence of apparatus design, in particular front boundary and exit-slit, on as measured test results, large-scale pullout tests were conducted (Raju, 1995) to enable comparison with results published by different researchers. The essential attributes of the apparatus are summarized in Table 1 and the details are given in the Raju (1995). The first comparison between results reported by Juran, et al. (1991), Palmeira (1987), and Raju (1995) are presented in Fig. 2. The mobilized interaction factor is presented with respect to the displacement of the clamped end ($d_c$). The geogrids used were Netlon SR2, Tensar SR2 and UX-1500. Netlon SR2 is a direct equivalent of Tensar SR2, while Tensar UX-1500 is slightly stronger. Soil and geogrid properties used in the laboratory tests are tabulated in Table 2 and Table 3, respectively. Vibratory compactor was used for compaction by Juran, et al. (1991); whereas Palmeira (1987) and Raju (1995) placed the soil by air-pluviation from a hopper. Properties of the geogrids and soils used in these tests were similar. The anchored lengths of the specimen were in the range of 0.91 m to 0.95 m. Hence, the only significant variability was in the pullout test apparatus and soil specimen preparation. The front boundary conditions of the test apparatuses were: steel surface of unspecified roughness (Juran, 1991), polyethylene sheets with grease in-between (Palmeira, 1987), and a low friction aluminum surface with $\delta = 12^\circ$ (Raju, 1995). The corresponding exit configurations were: a rigid sleeve type (Juran, 1991) and a no-sleeve type (Palmeira, 1987; and Raju 1995). It is important to note that a rough front boundary and no-sleeve combination was not utilized by any of the pullout apparatuses. The test results from the three apparatuses were summarized as $(F\alpha)_{mob}$ versus $d_c$ curves in Fig. 2. These curves, showing mobilization of normalized pullout resistance with displacement, compared well despite differences in the equipment. All curves exhibited an asymptotic value of maximum pullout resistance at large displacement. A second comparison was presented in Fig. 3 for stronger geogrids: Stratagrid 700 and Conwed G9027. The anchored lengths of the specimen were 0.91 to 0.92 m. The $(F\alpha)_{mob}$ versus $d_c$ curves for Stratagrid 700 at 4 kPa and 10 kPa applied normal stress showed a stiffer response with a slight but pronounced peak; but the residual value $(F\alpha)$ was similar to that of the 17 kPa test. It was recognized that at low applied stress, testing error due to slit friction could be more significant. Hence the geogrids were strain gauged. Observations deduced from the strain gauge response were similar. One can thus conclude that the test results of Raju (1995) were consistent. The stiffer and "pronounce peak" responses for the 4 kPa and 10 kPa tests were believed to be due to the
Fig. 2 Comparison of Pullout Test Results from Different Apparatus – Geogrid Group ‘A’

Fig. 3 Comparison of Pullout Test Results from Different Apparatuses – Geogrid Group ‘B’
ON LARGE-SCALE LABORATORY PULLOUT TESTING

Table 2  Properties of Soil Used in Laboratory Studies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>Leighton Buzzard sand 14/25</td>
<td>Uniform blasting sand</td>
<td>Uniform silica sand</td>
<td>Concrete sand</td>
<td>Poorly graded sand</td>
</tr>
<tr>
<td>$D_{50}$ (mm)</td>
<td>0.80</td>
<td>0.26</td>
<td>0.82</td>
<td>0.70</td>
<td>--</td>
</tr>
<tr>
<td>Unit weight (kN/m$^3$)</td>
<td>17</td>
<td>16.5</td>
<td>17.8</td>
<td>18.2</td>
<td>16.3</td>
</tr>
<tr>
<td>Particle range (mm)</td>
<td>0.6 to 1.18</td>
<td>0.1 to 1.18</td>
<td>0.6 to 1.18</td>
<td>$D_{10} = 0.31,$ $D_{50} = 2.6$</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 3  Properties of Geogrids Used in Laboratory Studies

<table>
<thead>
<tr>
<th>Geogrid</th>
<th>Open area (%)</th>
<th>Aperture (mm)</th>
<th>Thickness (mm)</th>
<th>Ultimate tensile strength (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MD</td>
<td>CMD</td>
<td>Rib</td>
</tr>
<tr>
<td>Tensar UX-1500</td>
<td>60</td>
<td>145</td>
<td>17</td>
<td>1.3</td>
</tr>
<tr>
<td>Netcon SR-2 &amp; Tensar SR-2</td>
<td>60</td>
<td>111</td>
<td>22.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Stratagrid 700 &amp; Conwed G-9027</td>
<td>46</td>
<td>58</td>
<td>24</td>
<td>1.7</td>
</tr>
</tbody>
</table>

inherent characteristics of soil and interface at very low stress. The 17 kPa test of Raju (1995) compared well with the 48 kPa test of Juran (1991). Good agreement between the measured initial responses was observed, although the maximum values manifested some slight differences, with Stratagrid (Raju, 1995) showing a slight strain-softening response. These results indicate that the difference in front boundary and exit-slit configuration of the three apparatuses has little significant influence when the scale of the testing is large.
The results reported by Wilson-Fahmy, et al. (1994) could be used to evaluate influence of $L_e$ on $(F\alpha)$. The apparatus dimensions in plan were 1.9 m x 0.91 m and 1.1 m deep, and with rigid sleeve as the exit-slit. Two different specimen lengths, 0.31 m and 0.92 m, of the same geogrid and embedded in the same soil were tested. The reported results, re-plotted in terms of $(F\alpha)$ and $(F\alpha)_{mob}$, are compared in Fig. 4. The clamp displacement, $d_c$, of the shorter specimen at pullout failure was 20 mm, which was smaller than that of the longer specimen. The mobilization of $(F\alpha)_{mob}$ also manifested a higher stiffness. This is expected as $d_c$ is partly due to extension of reinforcement. However, the $(F\alpha)$ value of the shorter specimen (0.31 m) was lower when compared to that obtained from a test with longer specimen (0.92 m), and the difference was about 15%. This is a rather unusual observation, and contrary to our expectation that shorter specimen has a higher $(F\alpha)$ value. To ensure the results of Wilson Fahmy, et al. are reliable, the $(F\alpha)_{mob}$ curve of a similar length of grid (1 m) obtained with a different apparatus (Collin and Berg, 1993) is also included in the comparison of Fig. 4. The dimensions of the apparatus used by Collin and Berg (1993) were 2.1 m x 0.9 m in plan and 0.5 m deep, and a rigid sleeve used at the exit-slit. The results of Collin and Berg (1993) manifested good agreement with the long specimen results of Wilson-Fahmy, et al. (1994). Therefore, the pullout apparatus of Wilson-Fahmy can be considered as reliable. But the lower $(F\alpha)$ value for the short specimen length remained unexplained.

![Fig. 4 Influence of Specimen Length on Pullout Test Results](image-url)
ON LARGE-SCALE LABORATORY PULLOUT TESTING

Some of the test results obtained for similar types of reinforcements embedded in similar soils showed good agreement despite the wide range of pullout box apparatuses used. Note that good agreement was restricted to specimen length exceeding 0.9m. However, the results of very short specimen length in a long pullout box was contrary to expectation despite that the same type of geogrid, the same soil, and the same apparatus were used. In an attempt to explain the above observation, a series of non-linear finite element analyses were conducted to study the influence of apparatus design on the \((F \alpha)\) value.

NON-LINEAR FINITE ELEMENT ANALYSIS

The influence of box depth, front boundary roughness and exit-slit configuration on the non-uniformity of the stress field within a pullout box can be effectively studied by a series of two dimensional non-linear finite element analysis. Note that non-uniformity in stress distribution may in turn affect the pullout resistance. The outcomes of the analysis can then be compared, qualitatively, with the experimental observations presented in the previous section. The soil was represented by 8-node 2D elements and the reinforcement by 3-node 1D elements. Interface elements are inserted between the reinforcement and surrounding soil to allow reinforcement pullout. The prescribed strength parameters for the analysis were:

\[
\text{soil:} \quad \phi = 35^\circ, \ c = 0.1 \text{ kPa}
\]
\[
\text{interface:} \quad \delta = 25^\circ, \ c = 0.5 \text{ kPa}
\]

where \(\phi\) is the friction angle of the soil, \(\delta\) is the interface friction angle, and \(c\) is the cohesion. The reinforcement was modeled as linear elastic and with a stiffness of 1000 kN/m. This is considered to be a reasonable stiffness for high strength geosynthetic. After the shear stress of an interface element attained its limiting value (as defined by \(\delta = 25^\circ\)), the shear stiffness was reduced to a small value (0.1% to 1% of the elastic value). This small fictitious stiffness at reinforcement pullout, however, was found to be inadequate in stabilizing the computation when reinforcement pullout was approached. The system was stabilized by extending the reinforcement outside the pullout box and supporting it by two soft elastic springs as illustrated in Fig. 5. The pullout force on the reinforcement, \(P\), is the applied force minus the spring reactions. \(P\) was obtained independently from the element located just outside the exit-slit. Before reinforcement pullout, the increment in spring force was essentially identical to the increment in \(P\). As failure was approached, the soft elastic spring played a significant role in absorbing the increment in applied force. A soft spring stiffness of 10 kN/m was found to be satisfactory and the analysis could proceed to pullout failure. A typical analysis comprised of three stages:

- created the box and gravity stresses
applied a pressure of 100 kPa on the top boundary

applied pullout force

The applied force was turned on incrementally with the magnitude of a force increment near pullout at about 1 kN. Due to the nature of the soil model used, the pressure of 100 kPa is simply a convenient value used for normalization, and does not limit the applicability of analysis results to specific $\sigma_{w}$ values of Eq. (2) and Eq. (2a). In calculating the $(F\alpha)$ and $(F\alpha)_{mob}$ values from the finite element results, the self weight of soil was included in the calculation of $\sigma_{w}$. The pullout box analyzed had a length of 1000 mm. Two box depths, 440 mm and 1100 mm, were investigated in the analyses. A sleeve length of 100 mm was adopted as a standardized value in the analysis.

Modeling of Exit Slit and Front Wall

Different exit-slit configurations were simulated by introducing a “front zone” with different (and adjustable) properties. A no-sleeve condition was simulated by prescribing a bonded condition with $\delta = 25^\circ$ (i.e. identical interface elements). A flexible sleeve condition was simulated by prescribing a “de-bonded” condition with $\delta = 0.05^\circ$. A small non-zero $\delta$ value was used to avoid numerical problems. A rigid sleeve condition was modeled by prescribing a “de-bonded” condition and providing fixity in
the vertical direction (i.e., allowing free movement only in the x-direction). A smooth front wall was simulated by providing fixity in the x-direction but allowing free movement in the y-direction. A rough front wall was modeled by attaching interface elements to the 2D elements. A strength parameter of $\delta = 25^\circ$ was used to simulate a high degree of roughness.

Soil Models

Since the analysis is not for a specific soil type, the computations were conducted with two different soil models: the Mohr-Coulomb elastic-plastic (MC) model and the Duncan-Chang (DC) model. Analysis using the MC model was conducted with the commercial program CRISP, whereas an in-house developed program NAGE (1995) was used for analysis based on the DC model. The MC model used a linear elastic formulation for pre-failure stress state but switched to plasticity formulation (with the failure surface taken as the yield surface) when failure criterion was satisfied. As such, the MC model gives dilatant kinematics at failure. Analysis could proceed to a few kN beyond pullout failure because of the non-zero residual stiffness of the interface elements. In the Duncan-Chang model, the Young’s modulus is given by:

$$E = K \left( \frac{\sigma_3}{p_a} \right)^n p_a (1 - S)^2$$

$$S = \frac{r_f (1 - \sin \phi)(\sigma_1 - \sigma_3)}{2c \cos \phi + 2\sigma_3 \sin \phi}$$

where $K$, $n$, $r_f$ are non-dimensional constants, and $p_a$ = atmospheric pressure in consistent unit. Note that the values of $r_f$ and $n$ are always less than or equal to unity. Hence, the Young’s modulus will reduce with the increase of deviatoric stress. Failure is modeled in Eq. (3) and Eq. (3a) because the Young’s modulus will approach zero as failure is approached. The kinematics at failure is non-dilatant. To avoid numerical problems as a soil element approached failure, a minimum Young’s modulus of 1 MPa was prescribed. This implies that, provided the load step is adequately small and the computation is performed with adequate precision, loading may proceed indefinitely. However, a kink followed by a “pronounced flattening” of the load displacement curve will be observed after pullout failure.

Analysis Results Based on MC Model

The analyses were conducted using an elastic Young’s modulus of 25 MPa and a Poisson’s ratio of 0.3 for soil, unless stated otherwise to the contrary in studying the influence of soil parameters. A relatively low Young’s modulus was used because, in the context of a MC model, it was a secant modulus.
The response during a simulated pullout test can be summarized by a \((F\alpha)_{mob}\) versus \(d_C\) curve which gives the mobilization of normalized pullout force with displacements. The \((F\alpha)_{mob}\) versus \(d_C\) curves of all three different exit-slit for a smooth front wall condition are compared in Fig. 6, where \(d_C\) is the reinforcement displacement at the exit. In calculating \((F\alpha)_{mob}\), the anchorage length of the specimen \((L_a)\) for a sleeved exit is shorter than the no-sleeve exit by 100 mm, the sleeve length. The difference between the three curves was slight. For the rigid sleeve case, a kink in the curve was observed at about 72 kN. However, a detailed examination of the shear stress of the interface element suggested that the interface shear stress was at the failure value when the pullout force was 68.8 kN, i.e., \(P_f = 68.8\) kN. Hence the “kink” only gave an approximate indication of pullout failure. Kinks in the \((F\alpha)_{mob}\) versus \(d_C\) curves may not be clearly observed for all cases. Hence pullout failures were determined by

![Fig. 6 (Fα) Mobilization Curves for Shallow Box with Smooth Front Wall](image-url)
ON LARGE-SCALE LABORATORY PULLOUT TESTING

examining the shear and normal stress of each and every interface element along the soil reinforcement interface. This enabled the determination of \( F_p \) to an accuracy of 1 kN. The \((F\alpha)\) values so obtained are summarized in Table 4. The difference between a no-sleeve and rigid sleeve condition was minimal, smaller than the achievable numerical accuracy. But the flexible sleeve condition gave about 10% higher pullout resistance. To further examine the influence of exit-slit configuration, the distributions of \( \sigma_n \) along the reinforcement for the three different exit-slit configurations were plotted in Fig. 7. Note that \( \sigma_n \) is the normal stress acting directly on the reinforcement and may be different from \( \sigma_{wo} \), the applied normal stress on the top boundary of the pullout box. For the no-sleeve and flexible sleeve configuration, the \( \sigma_n \) distributions at \( P = 0 \) were essentially uniform except in the immediate vicinity of the exit. The \( \sigma_n \) distribution for the rigid sleeve configuration was, however, highly non-uniform. There was a jump in the normal stress at the end of the rigid sleeve. This was because vertical displacement was prevented at the rigid sleeve despite that the soil was deformable. This led to very rapid change in the normal stress, which manifested numerically as a "jump", at the end of a rigid sleeve. As demonstrated in other sections, this was found to be characteristic of a rigid sleeve. The application of pullout force increased \( \sigma_n \) at the embedded end of the reinforcement, but reduced \( \sigma_n \) near the exit. Near pullout failure, the \( \sigma_n \) distributions

<table>
<thead>
<tr>
<th>Exit Configuration</th>
<th>Front Wall</th>
<th>Smooth</th>
<th>Rough</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-sleeve</td>
<td></td>
<td>0.447</td>
<td>0.46</td>
</tr>
<tr>
<td>Rigid-sleeve</td>
<td></td>
<td>0.453</td>
<td>0.46</td>
</tr>
<tr>
<td>Flexible-sleeve</td>
<td></td>
<td>0.493</td>
<td>0.477</td>
</tr>
</tbody>
</table>

Table 4 Comparison of \((F\alpha)\) Values

(a) 440 mm deep box

(b) 1100 mm deep box

<table>
<thead>
<tr>
<th>Exit Configuration</th>
<th>Front Wall</th>
<th>Smooth</th>
<th>Rough</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-sleeve</td>
<td></td>
<td>0.431</td>
<td>0.426</td>
</tr>
<tr>
<td>Flexible-sleeve</td>
<td></td>
<td>0.478</td>
<td>0.470</td>
</tr>
</tbody>
</table>
Fig. 7 $\sigma_n$ Distributions for Shallow Box with Smooth Front Wall
for all three exit configurations showed considerable non-uniformity. For the rigid sleeve condition, the jump in normal stress (at the end of the sleeve) and the high $\sigma_n$ gradient (over the rigid sleeve) still prevailed at reinforcement pullout. For the flexible sleeve and no-sleeve configurations, $\sigma_n$ reduced significantly near the exit. The higher $(F\alpha)$ value (but not $P_f$ value) for a flexible sleeve compared to a no-sleeve was partly due to the 100 mm zone (where $\sigma_n$ was lowest) being excluded from $L_e$. The non-uniformity in $\sigma_n$ over the anchored length was the least for the flexible sleeve configuration.

A similar comparison was made for rough front wall. The $(F\alpha)_{mob}$ versus $d_C$ curves for the three different exit configurations were similar (Fig. 8). The interaction factors (i.e., $(F\alpha)$ values) are compared in Table 4. The interaction factors of the no-

![Diagram](image_url)  

**Fig. 8** $(F\alpha)$ Mobilization Curves for Shallow Box with Rough Front Wall
sleeve and rigid sleeve configurations were essentially identical whereas the flexible sleeve configuration gave a slightly higher value. This observation is similar to that of smooth front wall. In fact the difference in \((F\alpha)\) values for all six cases were slight. The corresponding \(\sigma_n\) distributions are presented in Fig. 9. Despite the rough front wall, a jump in the normal stress was obtained. Due to the rough front wall, the \(\sigma_n\) distributions were non-uniform even without the application of pullout force. The \(\sigma_n\) distributions, however, changed with the application of pullout force. At reinforcement pullout, the \(\sigma_n\) distribution consisted of three zones. Zone-1 is defined approximately by \(x > 500\) mm, where \(x\) is the distance from the exit slit. For all three exit configurations, the \(\sigma_n\) curves were shifted upward relative to the initial distributions. Zone-2 is defined approximately by \(150\) mm \(< x < 500\) mm. The \(\sigma_n\) curves in this zone, for all three exit configurations, were shifted downward relative to the initial distribution. However, the "downward shift" for the no-sleeve configuration was slight. Zone-3 is defined by \(x < 150\) mm from the exit. For both the no-sleeve and flexible sleeve exit configurations, the \(\sigma_n\) curves were shifted upward relative to the initial values in this zone. This is believed to be due to constrained dilatancy effects, where the constraint was from the wall friction. For the rigid sleeve exit configuration, the jump in normal stress and high \(\sigma_n\) gradient were still the characteristic of this zone. This implied that the interference of the rigid sleeve was still dominant even at reinforcement pullout.

Since a rigid sleeve interferes considerably with the displacement field, the influence of a longer rigid sleeve length of \(180\) mm was also analyzed. The \((F\alpha)_{mob}\) versus \(d_e\) curves of the \(180\) mm sleeve case was nearly identical to the \(100\) mm rigid sleeve case.

Influence of Box Depth

To examine the influence of box depth, analyses for the following four cases were repeated for a \(1100\) mm depth box.

smooth front wall: no-sleeve and flexible sleeve

rough front wall: no-sleeve and flexible sleeve

The extra weight from the soil led to a slightly higher \(\sigma_{vo}\). The \((F\alpha)\) values were listed in Table 4. The influence of front wall roughness and exit slit configuration was again found to be slight, and the values were close to those of shallow box. The front wall condition and exit slit configurations also had little effect on the \((F\alpha)_{mob}\) versus \(d_e\) curves (Fig. 10). However, the curve for the flexible sleeve case manifested higher stiffness. This was consistent with the results of the shallow box. The \((F\alpha)_{mob}\) versus \(d_e\) curves of the deep box were compared with those of shallow box in Fig. 11. The deep box manifested a slight, but consistent, lower stiffness. The \(\sigma_n\) distributions, as presented in Figs. 12-13, are similar to those of the shallow box. However, the downward shift of the \(\sigma_n\) curves at pullout failure relative to the initial values
Fig. 9 $\sigma_n$ Distributions for Shallow Box with Rough Front Wall
occurred over a slightly larger zone. This explains the consistently lower \((F\alpha)\) values.

**Influence of Soil Parameters**

The rigid sleeve was found to induce considerable non-uniformity in \(\sigma_n\) distributions. To examine whether this is caused by the relatively low \(E\) value used for the soil continuum, the analysis was repeated with \(E = 100\) MPa. This is believed to be a very high secant Young's modulus. The pullout force and its mobilization were also
Fig. 11 Influence of Box Depth on (Fα) Mobilization
Fig. 12 $\sigma_v$ Distributions for 1100 mm deep Box with Smooth Front Wall
Fig. 13  $\sigma_n$ Distributions for 1100 mm deep Box with Rough Front Wall
near identical to the those obtained for $E = 25$ MPa. The trend of the $\sigma_n$ distributions as presented in Fig. 14 were similar to those obtained for $E = 25$ MPa. In particular, the jump in normal stress at the end of the rigid sleeve and the high $\sigma_n$ gradient over the rigid sleeve prevailed at both the initial condition ($P = 0$ case) and at reinforcement pullout. This implied that the rigid sleeve, which prevent vertical displacement at the sleeve location, induced a significant interference to the displacement and stress field for a wide range of realistic soil stiffness.

**Influence of Soil Model**

Since the choice of soil model for a non-linear finite element is not unique, some of the above analyses were repeated with the Duncan-Chang soil model. The parameters used are summarized in Table 5. The finite element results for a smooth front wall shallow box were compared in Fig. 15. The difference between the normalized force-displacement curves was slight. As discussed, the pullout resistances can be inferred from the “kinks” in the force displacement curves, or by comparing the shear stresses of the interface elements with the limiting value. The latter method, however, did not give improved accuracy due to the “noises” in the normal and shear stresses of the interface elements. A detailed examination suggested that the “kink” can be used to infer the pullout resistance with an accuracy of several kN. As such, one can conclude from the load displacement curve that the flexible sleeve configuration gave a pullout resistance approximately 12% higher than those of no-sleeve or rigid sleeve configuration. The $\sigma_n$ distributions for the three exit configurations and at pullout force of 40 kN were compared in Fig. 16. A jump in the axial stress at the end of the rigid sleeve was again obtained. The normal stress values over the rigid sleeve were however out-of-scale and hence not plotted in Fig. 16. It is evident that the application of pullout force changed the $\sigma_n$ distribution in a manner similar to that given by the MC model. Again the higher interaction factor (but not $P$) for the flexible sleeve case was partly due to the anchorage length not being located within the front 100 mm where $\sigma_n$ reduction was most significant.

**Table 5  Soil Parameters for Duncan-Chang Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ (deg)</td>
<td>35</td>
</tr>
<tr>
<td>$c$ (kPa)</td>
<td>0</td>
</tr>
<tr>
<td>$K$</td>
<td>1500</td>
</tr>
<tr>
<td>$n$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>0.80</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.30</td>
</tr>
</tbody>
</table>

146
440 mm deep box smooth front wall
high soil modulus, rigid sleeve

Smooth wall

Rough wall

$\sigma_n$ (kPa)

Distance (mm)

Fig. 14 Influence of Soil Modulus on $\sigma_n$ Distribution
Fig. 15 (Fo) Mobilization Curves for Shallow Box Using Duncan-Chang Soil Model

Fig. 16 \( \sigma_n \) Distributions for Shallow Box Using Duncan-Chang Soil Model
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The influence of front wall roughness for a deep box with a no-sleeve exit configuration was also investigated with the DC model. A consistent difference in the $(F_{\alpha})_{mob}$ versus $d_C$ curves was obtained (Fig. 17). The normalized pullout resistance of the rough front wall was about 15% lower than that of the smooth front wall. This is different from the finite element results based on MC model. This difference can be explained by the non-dilatant failure kinematics of the DC model. The rough wall reduced the normal stress acting on the reinforcement. The application of the pullout

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**Fig. 17** Influence of Wall Roughness for Deep Box with No-Sleeve, Duncan-Chang Soil Model
force, however, led to some increase in the vertical force acting on the reinforcement near the front wall. This mechanism was illustrated by the $\sigma_n$ distributions presented in Fig. 18. For a deep pullout box, the magnitude of the latter mechanism predicted by a DC soil model was not adequate to compensate for the lower initial normal stress. Hence, the interaction factor $(F\alpha)$ was lower compared to the smooth front wall. But the dilation of a MC model interacted with the wall roughness. This induced an extra increase in normal stress and hence the interaction factor $(F\alpha)$ so calculated was essentially the same as that of a smooth front wall.

**DISCUSSIONS**

Pullout test results are affected by boundary conditions of the pullout apparatus in a complicated way. However, comparison of large scale pullout test results obtained from different pullout apparatuses did not reveal significant dependence on apparatus design. The finite element analyses also indicated that the $(F\alpha)_{mob}$ versus $d_C$ curve and $(F\alpha)$ value were relatively insensitive to the range of box depth, wall boundaries and exit configurations in the non-linear finite element analysis. This qualitative, but important, finding was applicable for two different soil models. This finding adds confidence to the use of large-scale laboratory pullout testing in engineering design, and the use of non-linear finite element analysis in interpretation of pullout test results.

The finite element results suggested that influence of front wall roughness for the case of deep box appeared to be slightly dependent on the soil model assumed in the analysis. For the MC model, the $(F\alpha)$ value so deduced was essentially the same as that of a smooth wall condition. But the use of a non-dilatant DC model gave a $(F\alpha)$ value that is about 15% lower than the smooth wall case. No laboratory test results are available for this type of box for comparison. It is acknowledged that this "inconsistency" may be considered as small relative to the accuracy that can be achieved. But it is prudent to state that in the case of deep pullout box with no-sleeve, the influence of front wall roughness is uncertain.

It is important to note that, upon application of pullout force, $\sigma_n$ always became non-uniform. The $\sigma_n$ distribution was dependent on the exit configuration and roughness of the front wall. Provided $L_e$ was adequately large, the integrated average of $\sigma_n$ over $L_e$ would not manifest significant variation with sleeve configuration. Hence, despite the considerably non-uniformity of the stress field in a pullout box, the $(F\alpha)_{mob}$ versus $d_C$ curve and $(F\alpha)$ value were relatively insensitive to the range of box depth, wall boundaries and exit configurations in the non-linear finite element analysis. The use of a flexible sleeve led to a slightly stiffer $(F\alpha)_{mob}$ versus $d_C$ curves, a slightly higher $(F\alpha)$ value, and a $\sigma_n$ distribution with least non-uniformity.

For all the cases analyzed, a rigid sleeve led to a jump in normal stress at the end of the rigid sleeve and high $\sigma_n$ over the rigid sleeve. A careful scrutiny of all the
Fig. 18  $\sigma_n$ Distributions for Deep Box with No-Sleeve, Duncan-Chang Soil Model
Fig. 19 (Fα) Mobilization Curves for Short Specimen in Long Box

Fig. 20 σn Distribution for Short Specimen in Long Box
finite element results suggested that this was found to be a general characteristic of a rigid sleeve at all stages during a pullout test. Note that both soil models used permits yielding or stiffness reduction at high deviator stress. Hence the computed sharp stress jump was not "stress concentration" artificially induced by the limitation of the numerical models. However, it is also recognized that the sleeve in an actual pullout box may not have perfect rigidity, and thus the actual stress jump may be of a smaller magnitude.

The finite element results also provided a rational explanation for the rather unusual test result report by Wilson Falmey, et al. (1991): a very short specimen in a long box with a rigid sleeve had a lower normalized pullout resistance. For very short specimen length, \( L_e \), in a long pullout box, a substantial portion of the anchored length of the reinforcement may be in a region where \( \sigma_n \) is significantly lower than \( \sigma_{vo} \), which in turn may lead to lower pullout resistance. This explanation can be verified by analyzing the pullout testing of a short specimen (300 mm long anchored length) using the shallow box with rigid sleeve configuration. The \((F\alpha)_{mob}\) versus \(d_c\) curve for such an analysis is presented in Fig. 19. The \((F\alpha)\) value was 14% lower compared with the long specimen. The \(\sigma_n\) distribution was presented in Fig. 20. The \(\sigma_n\) value acting over the short anchored length was significantly lower than \(\sigma_{vo}\) and this led to a lower \((F\alpha)\) value.

Since \(\sigma_n\) will always become non-uniform after application of pullout force, the interpretation of local stress/strain or displacement measurement becomes problematic, as \(\sigma_n\) is an unknown that varies along the length of the reinforcement. This imposes severe limitations on how one can interpret internal measurements. Alternative, local pressure sensing films (Paikowsky and Hajduk, 1997) need to be incorporated with the internal measurements.

The limitations of the comparison of test results and the analysis also need to be recognized. The test results presented are somewhat limited relative to the complexity of variables involved. The soil model used in the analysis is rather simple and evidently cannot simulate the effects of strain softening along the interface and cyclic loading, both of which can increase the effects of non-uniform \(\sigma_n\) distribution. In this respect, it may be prudent to adopt a flexible exit slit configuration which gives the lower non-uniformity in \(\sigma_n\) distribution over the anchored length.

REFERENCES


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INFLUENCE FACTORS ON THE SHEAR MODULUS AND DAMPING OF RECLAIMED SOIL

L.K. Chien¹, G.L. Yeh² and Y.N. Oh²

ABSTRACT

In this study, the filled soil in Yun-Lin offshore area was adopted as testing samples. A successful hydraulic sand fill simulation method is developed in the laboratory to investigate packing properties of soil aggregates. The influence factors such as fines content, different sample preparation methods, different relative densities and confining pressures are taken into consideration. A series of resonant column test has been performed to study the shear modulus (G) and damping ratio (D) of the filled soil. The influence of effective confining pressure was discussed by using the $K_2$ value proposed by Seed and Idriss (1970). In order to evaluate the influence of fines content on maximum shear modulus of the filled soil, a fines content influence parameter $B$ is defined. With this parameter and on the basis of the evaluation method suggested by Hardin and Richart (1973), the influence of fines contents, effective confining pressure, void ratios and shear strain amplitudes on the maximum shear modulus of filled soil are discussed. The predicted results were shown to agree with the experiment results. The newly proposed evaluation method can be used to predict the maximum shear modulus of hydraulic filled soil.

INTRODUCTION

Hydraulic sand fill is one of the most important reclamation method in West Taiwan nearshore. The reclaimed land is important for the development of industrial estate (Sinotech, 1990). The fill materials for hydraulic sand fill were obtained from the seabed or river mouth by use of cutter and pump. According to Sladen and Hewitt (1989) study on Beaufort man-made island in Canada, the relative density of hydraulic filled soil ranges from 10% to 70%. The relative density is difficult to control and is normally below 50%. Thus, by understanding the mechanism involved during reclamation process, the reclamation in the coast line can be improved. The effects of earthquake, wave forces and pore pressure were considered as the factors affecting the

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stability of the nearshore reclamation area. Therefore, the purpose of this study is to evaluate the stability of the reclamation area by investigating the dynamic properties of the fill material.

In this study, a successful hydraulic sand fill method was developed to prepare the specimens and to simulate the packing properties of the fill material. Resonant column tests were conducted to discuss the dynamic properties of the fill material. Moist tamping method and multi-sieve pluviation through water method were used to prepare the specimens. The influences of relative density (void ratio), confining pressure and specimen preparation methods are discussed.

PREVIOUS RELATED STUDIES

The dynamic properties of soil are influenced by factors such as strain amplitudes, void ratio, overconsolidation ratio, fines content, effective confining pressure, soil aggregates, shear strain amplitude, specimen preparation method and etc. Hardin and Drenvich (1972) demonstrated the relationship between the shear stress and shear strain of sand to be a curve. When the strain increases, the slope of the curve decreases. Thus, the shear modulus decreases as the strain amplitude increases. Hall and Richart (1973) showed that the damping ratios increases as the shear strain amplitude increases for Ottawa sand.

Seed and Idriss (1970) proposed an equation for the relationship between shear modulus and confining pressure, and can be expressed as:

\[ G = 1000 \ K_2 \ (\sigma'_m)^{0.5} \]  

(1)

where \( \sigma'_m \) is the mean effective confining pressure in psf and \( K_2 \) is a parameter depending on the void ratio and stress strain of soil.

As demonstrated by Hardin and Richart (1963), for shear strain below \( 10^{-3}\% \), there is a maximum shear modulus for sand. The relationship between shear strain, void ratio and maximum shear modulus are illustrated by the following equation.

\[ G_{max} = A \ F(e) \ (\sigma'_m)^n \]  

(2)

where

\[ F(e) = (2.97 - e)^2 / (1 + e) \]  

(3)

for angular grained sand

and

\[ F(e) = (2.17 - e)^2 / (1 + e) \]  

(4)

for round grained sand

where, \( e \) is the void ratio, \( A \) is the dimensionless parameter of the soil and \( n \) is a parameter.
INFLUENCE FACTORS ON THE SHEAR MODULUS AND DAMPING

The influences on $K_2$ was illustrated by Hardin and Drenvich (1972). It was presented that, at low strain (shear strain $\leq 10^{-3}\%$), $K_2$ depends on the void ratio ($e$) of soil. At high strains (shear strain $\geq 10^{-1}\%$), $K_2$ is slightly influenced by vertical stress, but is essentially independent of $K_0$ (coefficient of lateral stress at rest), $\phi'$ (static strength parameter of soil in terms of effective stress) and $e$ (void ratio). For practical purposes, values of $K_2$ may be considered to be determined mainly by the $e$ (void ratio) or relative density and the strain amplitude of the motions. Thus, it was presented that, for relatively dense soil, the values of $K_2$ determined at low strains are typically in the range of 44 to 86.

Iwasaki and Tatsuoka (1977) obtained a maximum shear modulus for different clean sand and showed that it can be expressed as:

$$G_{\text{max}} = 900 F(e) \sigma'^m \text{ } (5)$$

where $F(e) = (2.17 - e)^2 / (1 + e)$.

The above equation shows that the shear modulus increases as the mean effective confining pressure increases. As shown by Hardin and Richart (1963), Iwasaki and Tatsuoka (1977), the parameter $n$ is influenced by the strain amplitude, and would increase as the strain amplitude increases. The values of $n$ ranges from 0.5 to 1.

Surendra, et. al. (1988) investigated the influence of cement content on shear modulus of Monterey No. 0 sand. The sand with 0%, 1%, 2%, 5%, and 8% of Portland cement added was adopted as test sample. Resonant column tests were performed with Drenvich Long-Tor Resonant Column Test Apparatus. The test results demonstrated that the shear modulus increases as the cement content increases.

Additionally, Sudrendra, et al (1988) investigated the influences of cement content on the damping ratios. The results demonstrated that the addition of small amount of cement would increase the damping ratios. For large amount of cement content, the damping ratios would decrease as the cement content increases.

Iwasaki and Tatsuoka (1977) adopted Iruma sand as test sample with 1 ~ 14% of fines added, and resonant column tests were conducted. The test results showed that the damping ratios decrease as the fine content increases.

The Institute of Harbor and Marine Technology (1986) in Taiwan conducted a series of cyclic dynamic triaxial test. The silty soil in the West Coast of Taiwan were adopted as test samples. The results showed that for non-plastic fines content of less than 10%, the shear modulus increases as the fines content increases. When the fines content is more than 10%, the shear modulus decreases as the fines content increases. When the fines content is about 20%, the shear modulus drops about 20%.
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Summarizing all the previous works and researches done, the dynamic properties are mostly discussed. But the influence factors are not studied. This paper describes the results of Seiken type resonant column test on the reclaimed soil in West Taiwan offshore area. The influence factors on dynamic properties of reclaimed soil are discussed. Newly developed empirical relations and their evaluation method are revealed.

EXPERIMENTAL PROCEDURE

Test Materials

In this study, the soil samples were obtained from Yun-Ling offshore area in west coast Taiwan. The index properties and grain size distribution of the sand used are shown in Table 1 and Fig. 1, respectively. Fines content were obtained from the soil samples passing through the #200 dry sieve. The weight of the specimen is provided as a component to control the fines content of the specimen. The sieving test and specific gravity test are conducted based on the ASTM-D452-85 and ASTM-D854-83 respectively. The maximum and minimum dry density tests are conducted based on the JSF-T26-81T of Japanese Soil Testing Manual (1979).

![Grain Size Distribution for Test Samples](image)

**Fig. 1** Grain Size Distribution for Test Samples
INFLUENCE FACTORS ON THE SHEAR MODULUS AND DAMPING

Table 1 Index Properties for Test Sample

<table>
<thead>
<tr>
<th>Properties</th>
<th>0 %</th>
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<th>16%</th>
<th>20%</th>
<th>30%</th>
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<td>Fines content</td>
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<td>Specific gravity, Gs</td>
<td>1.590</td>
<td>1.680</td>
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<td>1.790</td>
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<tr>
<td>Maximum dry density (g/cm³)</td>
<td>1.199</td>
<td>1.205</td>
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<td>1.214</td>
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<td>Minimum dry density (g/cm³)</td>
<td>0.164</td>
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<td>0.152</td>
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<td>Mean grain size diameter (mm), D₅₀</td>
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<td>0.165</td>
<td>0.159</td>
<td>0.157</td>
<td>0.155</td>
</tr>
<tr>
<td>Mean grain size diameter (mm), D₆₀</td>
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<td>0.129</td>
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<td>0.109</td>
<td>0.074</td>
</tr>
<tr>
<td>Mean grain size diameter (mm), D₃₀</td>
<td>0.100</td>
<td>0.074</td>
<td>0.068</td>
<td>0.046</td>
<td>0.025</td>
</tr>
<tr>
<td>Coefficient of uniformity, Cu</td>
<td>1.688</td>
<td>2.232</td>
<td>2.842</td>
<td>3.393</td>
<td>6.306</td>
</tr>
<tr>
<td>Coefficient of curvature, Cc</td>
<td>1.347</td>
<td>1.353</td>
<td>1.430</td>
<td>1.630</td>
<td>1.477</td>
</tr>
</tbody>
</table>

Sample Preparation

In this study, moist tamping method (M.T.), multi-sieve pluviation through water method (M.P.) and hydraulic sand fill method (H.F.) were adopted to prepare the specimen. The procedure of hydraulic sand fill method will be described in the following section.

In this study, hydraulic sand fill method was developed to simulate the filling procedure of the reclaimed materials on the reclamation area (Fig. 2). The simulation method is equipped with a submerged water pump (with a diameter of 50 mm and flow of 0.15 m³/min). The sand and water are uniformly mixed. Three pipes are connected to the pump. One is used as inlet for the sand water mixture. A total of six overflow pipes are connected to the acrylic tube to collect the excess mixture. The sand would then submerge and settle into the split mold. After the split mold is filled with sand, the specimen is kept for 10 minutes for the excess water to be drained away. This step is to avoid the specimen from being liquefied. The specimen prepared by hydraulic sand fill method has an initial relative density of 32% to 47%.

Test Procedure

The specimens were prepared by three specimen preparation methods (M.T., M.P., H.F.) with different fines content (0%, 10%, 16%, 20% and 30%). After the specimen was prepared, a vacuum of 19.6 kPa was applied on the top drain lead of the cell. The molds were then taken apart. Carbon dioxide was allowed to pass through the specimen for about 2 hours. To let the carbon dioxide to be fully dissolved, water was allowed to pass through the specimen for about 3 hrs. A back water pressure of
196 kPa was applied to the specimen in order to obtain a 95% saturation. Considering the in situ properties, different confining pressures (98 kPa, 147 kPa, 196 kPa and 294 kPa) were applied for different consolidation processes. According to Mulilis, et al. (1975), sand of similar density, after being consolidated for 24 hrs or 30 minutes would have similar strength. No aging process was observed. For the convenience of experiment, the consolidation process was proceeded for 30 minutes. Finally, the resonant column triaxial tests have been successfully used for testing specimens by using the resonant column apparatus Model DTC-158 (Fig. 3) developed by Seiken Inc., Japan.

**EXPERIMENTAL RESULTS AND ANALYSIS**

The main objective of this study is to investigate the influence factors on dynamic behavior of reclaimed soil at low amplitudes. In order to evaluate the influence of fines content on the dynamic properties of the specimen, moist tamping method is adopted to prepare the specimens with relative density of 40%, 60% and 80%. The specimens were then added with 0%, 10%, 16%, 20% and 30% of fines content. The fines are added during each layer of tamping. Resonant column tests were performed to find out the influences of fines content. Hydraulic sand fill method is also adopted to prepare the specimen to simulate the packing properties of the reclaimed soil. Multi-
pluviation through water method is also adopted to prepare the specimens (Table 2). Thus, the influences of different specimen preparation methods were discussed. The specimens were then prepared with different relative density and the tests were performed with different confining pressure. Finally, the influences of fines content on the dynamic properties of the reclaimed soil were also discussed. The sample preparation methods and experiment conditions are shown in Table 2.

**Relationship Between Fines Content and Void Ratios**

In this study, the weight of the specimen is provided as a component to control the fines content. Specimens with different fines content were prepared. The following equation shows the definition of fines content ($FC$).

$$FC(\%) = \frac{\text{Weight of fines of specimen}}{\text{Total weight of specimen}}$$  \hspace{1cm} (6)
Table 2 Sample Preparation Method and Experiment Conditions

<table>
<thead>
<tr>
<th>Specimen Preparation method</th>
<th>Set</th>
<th>Effective confining pressure</th>
<th>Initial relative density(%)</th>
<th>Fines content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moist tamping method (MT)</td>
<td>17</td>
<td>98kPa, 147kPa, 196kPa, 294kPa</td>
<td>40, 60, 80</td>
<td>0, 10, 16, 20, 30</td>
</tr>
<tr>
<td>Multi-sieve pluviation through water (MP)</td>
<td>8</td>
<td>98kPa, 147kPa, 196kPa, 294kPa</td>
<td>36~48</td>
<td>0~30</td>
</tr>
<tr>
<td>Hydraulic sand fill method (HF)</td>
<td>11</td>
<td>98kPa, 147kPa, 196kPa, 294kPa</td>
<td>32~47</td>
<td>0~8</td>
</tr>
</tbody>
</table>

The relationships between fines content, maximum void ratio and minimum void ratio are shown in Fig. 4. When the fines content is less then 20%, the maximum void ratio does not change accordingly to fines content. This is due to the arch effects of loose sand (Lambe and Whitman, 1987). For minimum void ratio, it would decrease as the fines content increases. This is due to the replacement of fines in the voids.

![Graph showing the relationship between fines content and void ratio](image)

**Fig. 4 Relationship Between Fines Content and Void Ratio**
INFLUENCE FACTORS ON THE SHEAR MODULUS AND DAMPING

Relationship Between Fines Content and Specific Gravity

The influence of fines content on the specific gravity can be obtained from the specific gravity test. For 0% of fines content, the specific gravity of the clean sand is 2.691. While, the specific gravity of fines is 2.717. By regressing the experimental data (Fig. 5), the relationship between fines content and specific gravity can be demonstrated by the following equation.

\[ G_s = 0.000255 \times FC \% + 2.692 \]  \hspace{1cm} (7)

where \( FC \) is the fines content and \( G_s \) is the specific gravity.

Relationship Between Fines Content and Index Properties of Soil

The relationship between fines content and the related soil parameters are listed on Table 1. As shown in the table, the coefficient of uniformity \( (C_u) \) increases as the fines content increases.

Influence of Shear Strain Amplitude

The experiment was proceeded with different confining pressures. The typical results presented in Fig. 6 clearly show that the shear modulus of the reclaimed soil decreases as the shear strain amplitude increases. This is mainly due to the nonlinearity stress-strain relationship of soils. As shown in Fig. 7, the damping ratio increases as the shear strain increases, and the increase is caused by energy absorption due to particle rearrangement. Importantly, it is observed that for strains less than \( 5 \times 10^{-4} \%), the shear modulus \( (G) \) values remain constant, hence the maximum shear modulus \( (G_{max}) \) is defined.

![Fig. 5 Relationship Between Fines Content and Specific Gravity](image-url)
Influence of Confining Pressure

The influence of confining pressure on the reclaimed soil can be shown by Fig. 6 and Fig. 7. As shown in Fig. 6, the shear modulus of the reclaimed soil increases as the effective confining pressure increases, while the damping ratio of the reclaimed
soil decreases as the effective confining pressure increases (Fig. 7). As the effective confining pressure increases, the specimen becomes denser. Thus, the number of contact points among the soil aggregates increases. This would allow the stress wave to propagate with a higher velocity. Therefore, the shear modulus would increase as the effective confining pressure increases.

To investigate the influence of effective confining pressure on the reclaimed soil with different fines content, the evaluation method suggested by Seed and Idriss (1970) was used, and as expressed in Eq. (1).

As shown in Fig. 8, for 10% of fines content, there is a maximum value for $K_2$. As the fines content increases to more than 10%, $K_2$ would decrease. As for the relation with shear strain, $K_2$ decreases as the shear strain increases.

Taking the corresponding value of shear strain at $5\times10^{-4}$% with $K_2$ adopted in Fig. 8, a $K_{2\max}$ can be obtained, and the results are illustrated in Fig. 9. As shown in the figure, for relative density of 40%, $K_{2\max}$ does not have large variation. As for 60% and 80% of relative density, $K_{2\max}$ increases as fine content increases, beyond 10% fines content $K_{2\max}$ decreases as the fine content increases.

![Graph](image_url)

**Fig. 8** Relationship Between $K_2$ and Shear Strain under Different Fines Content
Influence of Relative Density (Void Ratio) on the Dynamic Properties of Reclaimed Soil

The tests were conducted under the same effective confining pressure (98 kPa) to investigate the influence of different relative densities on the dynamic properties of the reclaimed soil. As shown in Fig. 10, the shear modulus increases as the relative density are increased. In general, the shear modulus increases as the void ratios are decreased. At higher density, the contacts between the sand grains are increased and thus the velocity of shear wave propagation increases. On the other hand, damping ratios are not significantly influenced by the relative density. For lower relative density, greater damping ratios can be observed, as shown in Fig. 11.
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![Graph showing the relationship between shear modulus G (MPa) and shear strain (%).](image)

Fig. 10 Relationship Between Shear Modulus and Shear Strain with Different Relative Density

![Graph showing the relationship between damping ratio (%) and shear strain (%).](image)

Fig. 11 Relationship Between Damping Ratios and Shear Strain with Different Relative Density

Influence of Fines Content on the Shear Modulus of Reclaimed Soil

In this study, to investigate the influence of fines content on the shear modulus, specimens with different fines content were prepared and a relative density of 60% was used to control the specimen. Resonant column tests were conducted with an effective
confining pressure of 98 kPa. As shown in Fig. 12, the shear modulus decreases as the shear strain increases. Under the same strain amplitudes, when there is 10% of fines content, the shear modulus has the greatest values. For fines content with 0%, 16%, 20% and 30%, the shear modulus decreases. This is due to the contact surfaces among the soil aggregates. When the fines content is 10%, the void ratios were filled with fines, but the contact surfaces among the soil aggregates were not reduced. Thus, the stress waves were transmitted with maximum velocity and greatest shear modulus were obtained. When the fines content exceeds 10%, the increase of fines content would influence the structures of soil aggregates. The voids are filled by fines and would separate the contact surfaces between soil aggregates. Thus, the shear modulus would reduce.

Influence of Fines Content on Damping Ratio on the Reclaimed Soil

The influences of fines content on damping ratios are not distinctive. Generally, the damping ratios increase as the shear strain increases, as shown in Fig. 13.

Influence of Fines Content on $G_{\text{max}}$

In this study, the influences of effective confining pressure on the $G_{\text{max}}$ of the reclaimed soil with different fines content were discussed. The specimens were prepared with a relative density of 60%. The relationship between $G_{\text{max}}$ and effective confining pressure with different fines content is shown in Fig. 14. As shown in Fig. 14, $G_{\text{max}}$ increases as the effective confining pressure increases. The increase of effective confining pressure would make the specimen denser. For 10% of fines content, there is a greatest value for the $G_{\text{max}}$.

To investigate the influence of fines content on $G_{\text{max}}$, the relationship between $G_{\text{max}}$ and different fines content with different relative density were discussed (Fig. 15). For specimen with relative density of 40% with more than 10% of fines content, the variation is not distinctive. For specimens with relative density of 60% and 80%, the influence of fines content is observed. For fines content less than 10%, $G_{\text{max}}$ increases as the fines content increases. For fines content more than 10%, $G_{\text{max}}$ decreases as the fines content increases. As shown in the figure, for 10% of fines content, $G_{\text{max}}$ reaches the maximum value.

In this study, the influence factors (strain amplitudes, effective confining pressure, void ratios and fines content) on the maximum shear modulus are discussed. Using the evaluation method proposed by Hardin and Richart (1963), the influence factors are discussed by using Eq. (4).
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Fig. 12 Relationship Between Shear Modulus and Shear Strain with Different Fines Content (Moist Tamping Method)

Fig. 13 Relationship Between Damping Ratio and Shear Strain with Different Fines Content (Moist Tamping Method)

Fig. 14 Relationship Between $G_{max}$ and Effective Confining Pressure with Different Fines Content (Moist Tamping Method)
In this study, a fines content influence parameter $B$ is defined as:

$$B = \frac{A_f}{A_0}$$  \hspace{1cm} (8)

where $A_f$ is the dimensionless parameter of the reclaimed soil with fines content; $A_0$ is the dimensionless parameter of the reclaimed soil with 0% of fines content. $A_f$ and $A_0$ are defined based on Eq. (2), which was proposed by Hardin and Richart (1963).

Using the above equation, $G_{\text{max}}$ with different fines content can be simplified by the following equation:

$$G_{\text{max}} = A_0 B F(e) \left( \sigma'_m \right)^n$$  \hspace{1cm} (9)
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The sand used in this study is classified as angular grained sand. Therefore, Eq. (5) can be used to evaluate the influence of void ratio on $G_{\text{max}}$. The relationship between effective confining pressure and $G_{\text{max}}/F(e)$ is shown in Fig. 16.

Under different fines content, the equations showing the relation curve between effective confining pressure and $G_{\text{max}}/F(e)$ can be represented by the following equations.

\[
FC = 0\% , \ G_{\text{max}} = 204.0 \ F(e) \ (\sigma'_m)^{0.58} \tag{10}
\]

\[
FC = 10\% , \ G_{\text{max}} = 227.4 \ F(e) \ (\sigma'_m)^{0.54} \tag{11}
\]

\[
FC = 16\% , \ G_{\text{max}} = 189.0 \ F(e) \ (\sigma'_m)^{0.54} \tag{12}
\]

\[
FC = 20\% , \ G_{\text{max}} = 163.4 \ F(e) \ (\sigma'_m)^{0.54} \tag{13}
\]

\[
FC = 30\% , \ G_{\text{max}} = 139.6 \ F(e) \ (\sigma'_m)^{0.61} \tag{14}
\]

Considering the influence of void ratios and effective confining pressure on the maximum shear modulus, the parameter $A_0$ and $n$ can be related with different fines content. The parameter $n$ does not have large variation, thus the average value of 0.56 for $n$ is adopted for this study. Combining the fines content influence parameter $B$ as in Eq. (8), the maximum shear modulus for different fines content is illustrated as the following equation.

\[
G_{\text{max}} = 204 \ B \ F(e) \ (\sigma'_m)^{0.56} \tag{15}
\]

![Fig. 16 Relationship Between Effective Confining Pressure and $G_{\text{max}}/F(e)$ with Different Fines Content (Moist Tamping Method)](image)
The relationship between fines content influence parameter $B$ and fines content is shown in Fig. 17. As shown in the figure, there is a turning point for fines content influence parameter when the fines content is 10%. Regressing the curves relating fines content and fines content influence parameter, the related equations can be shown as follows.

- For soil samples prepared by moist tamping method, the related equations can be illustrated in Eq. (16) and Eq. (17).

For $0\% < FC < 10\%$,

$$B = 1 + 0.0216 \ (FC) - 0.001 \ (FC)^2$$  \hspace{1cm} \text{(16)}

For $10\% < FC < 30\%$,

$$B = 1.603 - 0.0573 \ (FC) + 0.001 \ (FC)^2$$  \hspace{1cm} \text{(17)}

where $FC$ is the fines content.

- For soil samples prepared by multi-sieve pluviation through water method, the related equations can be illustrated in Eq. (18).

For $0\% < FC < 30\%$,

$$B = 0.998 - 0.116 \ (FC) + 0.002 \ (FC)^2$$  \hspace{1cm} \text{(18)}

- For soil samples prepared by hydraulic sand fill method, the related equations can be illustrated in Eq. (19).

For $0\% < FC < 8\%$,

$$B = 0.999 - 0.0054 \ (FC) - 0.0016 \ (FC)^2$$  \hspace{1cm} \text{(19)}

**Influence of Different Specimen Preparation Methods.**

By using different specimen preparation method, the influence of sample preparation method is discussed. For the specimen prepared by hydraulic sand fill method and multi-sieve pluviation through water method, the fines content and the relative density are not easily controlled. Thus, the specimen is prepared with 0% fines and a relative density of 40%, to examine the influence of specimen preparation method.

As shown in Fig. 18, the shear modulus for the specimen prepared by hydraulic sand fill method and multi-sieve pluviation through water method have similar values. As for the specimen prepared by moist tamping method, there is a greatest value of
shear modulus. As shown in Fig. 19, the specimen prepared by moist tamping method has the greatest damping ratio.

For the specimen prepared by multi-sieve pluviation through water method and hydraulic sand fill method, the values of $B$ decrease as the fines content increases (Fig. 17). It may be noted that there is a maximum value of $B$ at 10% of fines content for the specimen prepared by moist tamping method. A definite decrease of $B$ value with the specimen prepared by hydraulic sand fill method is observed. The tendency of decrease in $B$ value with fines content experimented by specimens prepared by multi-sieve pluviation through water method, is slower than other specimen preparation methods.

Once $B$ value is known, $G_{\text{max}}$ can be evaluated from above proposed equation, with measured values as shown in Fig. 20, test results showed that the predicted results match with the experiment results. Therefore, this proposed method by the authors can be used to evaluate the maximum shear modulus of reclaimed soil.

![Graph showing relationship between parameter $B$ and fines content with different specimen preparation methods.](image)

**Fig. 17** Relationship Between Parameter $B$ and Fines Content with Different Specimen Preparation Methods
Fig. 18  Relationship Between Shear Modulus and Shear Strain with Different Specimen Preparation Methods

Fig. 19  Relationship Between Damping Ratio and Shear Strain with Different Specimen Preparation Method

Fig. 20  Comparison of Measured Maximum Shear Modulus ($G_{\text{max}}$) and Tested Maximum Shear Modulus ($G_{\text{max}}$) under Different Specimen Preparation Methods
INFLUENCE FACTORS ON THE SHEAR MODULUS AND DAMPING

CONCLUSIONS

This study shows the influences on the dynamic properties of reclaimed soil in west Taiwan. The results show that for specimens with 0%, 16%, 20% and 30% of fines content, the dynamic shear modulus decrease as the fines content increases. The dynamic shear modulus of reclaimed soil has the maximum values for 10% of fines content under the same strain amplitudes. The damping ratios are found to be less influenced by fines content. The influences of relative density (void ratio), effective confining pressure, specimen preparation methods are also discussed.

The dynamic shear modulus of reclaimed soil increases as the relative density increases. The test results illustrated that the shear modulus increases as the effective confining pressure increases. For the specimen prepared by moist tamping method, for fines content less than 10%, $G_{\text{max}}$ increases as fines content increases, for fines content exceeding 10%, $G_{\text{max}}$ decreases as fines content increases. For the specimen prepared by multi-sieve pluviation method and hydraulic sand fill method, $G_{\text{max}}$ decreases as fines content increases.

Using the method suggested by Seed and Idriss (1970), the $K_2$ value is used to discuss the influence of effective confining pressure. As shown in the results, for 10% of fines content, there is a greatest value for $K_2$. As the fines content increases more than 10%, $K_2$ would decrease. Taking the corresponding value of shear strain at $5 \times 10^{-4}$% to $K_2$, a $K_{2\text{max}}$ can be obtained, and the results showed that for 40% of relative density, $K_{2\text{max}}$ does not have great difference. As for 60% and 80% of relative density, $K_{2\text{max}}$ increases as fine content increases.

In this study, a fines content influence parameter $B$ is defined to evaluate the influence of fines content with different proposed methods. Newly proposed evaluation method for maximum dynamic shear modulus is based on reliable resonant column test results and are convenient to use for the reclaimed soil.

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ELECTRO-OSMOTIC CONSOLIDATION BEHAVIOR OF TWO ONTARIO CLAYS

J.Q. Shang\(^1\) and K.S. Ho\(^2\)

ABSTRACT

Results of laboratory tests on electro-osmotic consolidation of two Ontario clays were studied, including the consolidation pressure induced by electro-osmosis from the theoretical computation and experimental measurement, change of soil void ratio, and rate of electro-osmotic consolidation. It was found that the final electro-osmotic consolidation settlement resembled that induced by mechanical consolidation, whereas the rate of electro-osmotic consolidation was considerably lower. Therefore, electro-osmotic consolidation may achieve similar effects as mechanical consolidation at longer time period. These factors should be considered in the design of electro-osmotic consolidation of clayey soils.

INTRODUCTION

Improvement and stabilization of soft clayey soils remain a challenge facing geotechnical engineers today. For example, landslides that frequently occurred in the area of Ottawa Valley, Canada, are well known to many geotechnical engineers. There was a recent landslide at Lemieux, Ontario on June 20, 1993 (Evans and Brooks, 1994). The indirect and direct costs associated with the Lemieux landslide were estimated as Cdn $2.5 million. The strengthening of soft clays is often necessary to ensure adequate slope stability in excavation. During the construction of the new St. Clair River Tunnel between Sarnia, Ontario and Port Huron, Michigan, monitoring of the approach cut in Sarnia identified slope movements with various rates and magnitudes. Serious concerns lead to an intensive geotechnical investigation (Becker, et al., 1996).

Over the years, electro-osmotic consolidation has been applied in projects such as stabilization of earth dams, railway embankments and slopes, and strengthening of pile foundations. A review of the principles and case histories of electro-osmotic consolidation was presented by Casagrande (1983). Many successful projects were reported in the literature, including the West Branch Dam, Mahoning River, Ohio (Fetzer, 1967), Trans-Canada Highway Bridge, Little Pic River, Ontario (Casagrande, et al., 1961), Kooteney Canal, British Columbia (Wade, 1976), Canadian Pacific Railway Cut, Revelstoke, British Columbia (Casagrande, et al., 1981), excavation for Revelstoke Dam, British Columbia

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(Casagrande, 1983), Big Pic River Bridge, Ontario (Soderman and Milligan, 1961), and a field pilot test at Gloucester, Ontario (Lo, et al., 1991a). However, the design of electro-osmotic consolidation remains semi-empirical. Particularly the electro-osmotic consolidation behavior of clayey soils, namely, the magnitude and rate of settlement, has not been systematically studied.

BACKGROUND

The principles of electro-osmotic consolidation are well established. When a direct current (DC) electric field is applied onto a soil mass, soil pore water is attracted to the cathode of negative polarity. The one-dimensional governing equation of electro-osmotic consolidation can be expressed in a diffusion equation in terms of a dummy variable, $\xi$ (Esrig 1968):

$$\frac{\partial \xi}{\partial t} = C_w \left( \frac{\partial^2 \xi}{\partial x^2} \right)$$ (1)

where $t$ = time (s), $x$ = spatial coordinate and $C_w$ = coefficient of electro-osmotic consolidation (m$^2$/s). The excess pore pressure, $u_e(x, t)$, generated by electro-osmosis at time $t$ and location $x$, can be obtained after solving the dummy variable $\xi$ in Eq. (1),

$$u_e(x, t) = \xi(x, t) - (k_e/k_h) \gamma_w V(x)$$ (2)

where $k_e$ (m$^2$/s/V) = coefficient of electro-osmotic permeability, $k_h$ (m/s) = coefficient of hydraulic conductivity, $\gamma_w$ (kN/m$^3$) = unit weight of water, and $V(x)$ (volt) = electrical potential at a distance $x$ (m) to the cathode at which $V(0) = 0$. Under the condition of cathode open and anode closed to drainage, electro-osmosis induces negative pore pressure that is linearly with the maximum suction at the anode and zero at the cathode. The increase in effective stresses due to electro-osmosis governs the consolidation process, similar to that of mechanical consolidation. When the electric field establishes the equilibrium of negative pore pressure in soil mass, the increase in the effective stress, $\Delta \sigma'$, can be expressed as:

$$\Delta \sigma' = -u_{so} = (k_e/k_h) \gamma_w V(x)$$ (3)

In the design of electro-osmotic consolidation, two issues must be addressed, namely (1) the consolidation pressure generated by electro-osmosis as it is related to the applied electric field and soil properties, and (2) time rate of electro-osmotic consolidation settlement. If the physical-chemical properties of the soil remain unchanged with time and there are no electrode reactions, the coefficient of electro-osmotic consolidation, $C_w$, would be identical to the coefficient of consolidation, $C_v$, under mechanical loading. However, it has been found that electrode reactions and associated physical-chemical changes are predominant over the period of electro-osmotic consolidation (Casagrande, 1983; Mitchell, 1993; and Shang, et al., 1996), which generated effects such as pH gradient, increase of pore water conductivity and changes of soil Atterberg limits. Therefore, an evaluation of $C_w$ is necessary for design purposes.
ELECTRO-Osmotic Consolidation Behavior of Two Ontario Clays

SITE LOCATION AND SOIL CONDITIONS

Two clayey soils were recovered from Gloucester and Wallaceburg in Ontario. The Gloucester National Test Site is operated by the National Research Council of Canada. The subsoil at the site consists of a 20 m thick deposit of Champlain Sea clay which covers a large portion of the lowlands of the St. Lawrence River and Ottawa River valleys. The soil is classified as a soft to very soft, very sensitive silty clay with geotechnical, mineralogical and chemical properties that have been investigated extensively (Boozuk and Leonards, 1972; Lo, et al., 1976; and Shang, et al., 1994). Shelby tube (152.4 mm) samples were recovered from the depth between 2 m and 6 m. The field vane strength of the clay was about 12 kPa with water content varying from 60% to 100%. The average liquid and plastic limits were 48% and 24%, respectively. The sensitivity of the clay was about 100 with an overconsolidation ratio of 1.5. The Wallaceburg site is located about 2 km east of Wallaceburg. The thick clay deposits of this area are within the St. Clair clay plains and are generally classified as normally consolidated, water-laid tills of the late Wisconsin substage of glaciation (Lo and Becker, 1979). The subsoil condition at the site consists of 0.15 m of topsoil, 2.9 m of stiff to very stiff brown silty clay crust, and 18 m of firm to soft grey silty clay. The 304.8 mm cube block samples were recovered from the 9 m thick upper firm to soft silty clay stratum. The field vane strength of the clay was about 22 kPa with an average water content of 38%. The average liquid and plastic limits were 45% and 21%, respectively. The sensitivity of the clay was about 5 with an overconsolidation ratio of 9.

EXPERIMENTAL SET-UP AND PROCEDURE

Two testing cells, namely, cells A and B, were developed at the University of Western Ontario. Cell B was initially developed by the second author during his service with Golder Associates, Ltd. in Mississauga, Ontario, Canada and was further developed at the University of Western Ontario. The schematics of the cells are shown in Fig. 1 and Fig. 2. Detailed descriptions of the apparatus have been discussed elsewhere (Lo, et al., 1991b; and Shang, et al., 1996).

As shown in Fig. 1, cell A simulates one-dimensional compression by using a confining cell. The pore pressure along the axis of the soil specimen was measured through four pore pressure probes. The design has the advantage of being able to obtain pore pressure distribution along the soil sample. However, since the negative pore pressure induces suction and shrinkage of the soil specimen, a gap between the soil specimen and the cell wall develops soon after the application of a DC voltage.

Cell B adopted the design of a triaxial cell, as shown in Fig. 2. The soil specimen can be consolidated under the in-situ $k_o$ condition before testing. Back-pressure can be applied through an independent pressure system to ensure saturation of the soil sample and to simulate the in-situ effective stress condition. The pore pressure was measured at the anode,
as shown in Fig. 2. The design avoids the soil disturbance by inserting the pore pressure probes into the soil, which becomes considerable when large vertical deformation occurs during consolidation. The effect of lateral deformation can be accounted for using the settlement (vertical deformation) and volume change data.

The electrodes used in both cells were made of porous brass plates installed at the top and bottom of the soil specimen. All tests were conducted after the soil sample had been mechanically consolidated at its equivalent in-situ effective overburden pressure and the excess pore pressure dissipated more than 95%. Therefore, the soil deformation which occurred during a test was attributed primarily to electro-osmotic consolidation. During an electro-osmotic test, the anode was closed and the cathode opened to drainage (back-pressure line) to comply with the boundary condition required for the generation of the negative pore pressure. The settlement (vertical deformation) of the soil specimen, pore pressure, volume change, applied voltage and current were measured during the experiments.
Fig. 2 Electro-osmotic Testing Cell B
RESULTS AND DISCUSSION

Table 1 summarizes the results of the electro-osmotic tests conducted on the Gloucester and Wallaceburg clays. The changes of void ratio after electro-osmotic consolidation, \(\Delta e\) (column (6), Table 1) were estimated from the soil settlement and volume change after testing. The hydraulic permeabilities and electro-osmotic permeabilities (columns (8) and (9), Table 1) are considered suitable for electro-osmotic consolidation. To generate meaningful electro-osmotic pore pressure, the ratio of the electro-osmotic and hydraulic permeabilities, \(k_e/k_h\), should be at least in the order of 0.1 m/V, which suits well with the measured \(k_e/k_h\) ratios (in the range 1.4 to 4.1 m/V). The average effective stress increases were computed from Eq. (3) and presented in column (10-A), whereas the measured pore pressures are presented in column (10-B), Table 1. For comparison, the ratio of the measured and computed values are presented in column (10)-C with the average of 1.4 and standard deviation of 0.5. This suggests that the calculated effective stress increases are statistically higher than the measured negative pore water pressures. The discrepancy is attributable to the following factors: (1) voltage loss at electrode-soil contacts due to activation and concentration polarization in the vicinity of electrodes (Oldham and Myland, 1994), which is considered as the predominant cause. Casagrande (1983) suggested that an empirical efficiency factor ranging from 0.6 to 0.9 should be used in design to take these effects into account; (2) variation of the electroosmotic and hydraulic permeability of soil due to changes of soil properties. It was observed that a dense, stiff crust formed at electrode-soil contacts after approximately 24 hours of treatment, which impeded water transport significantly; and (3) errors involved in pore pressure measurements. The pore pressure transducers were interrupted considerably due to desiccation, gas generation and formation of the dense crust at electrode-soil contacts. The problem became more significant at higher applied voltage gradients (Shang, et al., 1996). Pressure probes used in cell A were installed along the wall and at least 10 mm away from the electrode plates, which reduced the problem to some extent. However, the pore pressure probes were bent due to soil deformation, which could induce errors of measurement. The problem became evident when large settlement and volume change were generated in the Gloucester clay tests.

Figures 3 and 4 present typical settlements versus logarithm time for the Gloucester and Wallaceburg clays, respectively. The settlement curves for the rest of the tests can be found in Ho (1990) and Shang, et al. (1996). The coefficient of electro-osmotic consolidation, \(C_w\), can be experimentally determined from the rate of electro-osmotic settlement. In particular, at the time of 50% consolidation, \(C_w\) can be computed from Casagrande’s logarithm of time fitting method. The results of the computation are summarized in column (11), Table 1. In some tests, the applied voltage was changed or the tests were terminated before the intersection of the tangent and the asymptote could be clearly defined, as shown on the settlement curves marked “?” in Fig. 3(a) and Fig. 4(a). Therefore, the changes of void ratio reported in column 6, Table 1, may not always represent those at 100% consolidation. Similarly, the calculation of \(C_w\) in some cases involved uncertainties from the estimated asymptotes as illustrated in Fig. 3(a) and Fig. 4(a) (dash-lines).
Table 1 Summary of Test Results

<table>
<thead>
<tr>
<th>Soil</th>
<th>Test No.</th>
<th>Cell</th>
<th>$H_0$ (mm)</th>
<th>$e_o$</th>
<th>$\Delta e$</th>
<th>$V_e$ (volt)</th>
<th>$K_h \times 10^3$ (m/s)</th>
<th>$K_e \times 10^3$ (m$^2$/V)</th>
<th>$\sigma_{os}$ (kPa)</th>
<th>$-U_{os}$ (kPa)</th>
<th>$\sigma_{os}/(-U_{os})$ (m$^3$/a)</th>
<th>$C_w$ (m$^3$/a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gloucester</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GV4A</td>
<td>A</td>
<td>102</td>
<td>2.71</td>
<td>0.124</td>
<td>3</td>
<td>0.83</td>
<td>1.8</td>
<td>31.9</td>
<td>15</td>
<td>2.1</td>
<td>4.71</td>
<td></td>
</tr>
<tr>
<td>GV4B</td>
<td>A</td>
<td>101</td>
<td>1.92</td>
<td>0.082</td>
<td>1.5</td>
<td>0.83</td>
<td>1.8</td>
<td>16.0</td>
<td>8</td>
<td>2.0</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>GV8</td>
<td>A</td>
<td>200</td>
<td>2.71</td>
<td>0.137</td>
<td>3</td>
<td>0.83</td>
<td>1.8</td>
<td>31.9</td>
<td>13</td>
<td>2.5</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>B</td>
<td>99</td>
<td>1.76</td>
<td>0.113</td>
<td>4</td>
<td>1.91</td>
<td>4.56</td>
<td>46.8</td>
<td>45</td>
<td>1.0</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>B</td>
<td>100</td>
<td>1.65</td>
<td>0.138</td>
<td>6</td>
<td>1.91</td>
<td>4.14</td>
<td>63.8</td>
<td>61</td>
<td>1.0</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>B</td>
<td>147</td>
<td>2.17</td>
<td>0.233</td>
<td>6</td>
<td>2.36</td>
<td>4.59</td>
<td>57.2</td>
<td>66</td>
<td>0.9</td>
<td>1.8</td>
<td></td>
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<tr>
<td>Wallaceburg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>WH3</td>
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<td>76</td>
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<td>0.024</td>
<td>2</td>
<td>1.7</td>
<td>7</td>
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<td>25</td>
<td>1.6</td>
<td>1.12</td>
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<td>0.014</td>
<td>2</td>
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<td>7</td>
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<td>50</td>
<td>1.6</td>
<td>1.48</td>
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<td>A</td>
<td>229</td>
<td>1.03</td>
<td>0.010</td>
<td>2</td>
<td>1.7</td>
<td>7</td>
<td>80.8</td>
<td>51</td>
<td>1.6</td>
<td>2.63</td>
<td></td>
</tr>
<tr>
<td>WV4</td>
<td>A</td>
<td>101</td>
<td>1.06</td>
<td>0.016</td>
<td>2.5</td>
<td>3</td>
<td>7</td>
<td>28.6</td>
<td>30</td>
<td>1.0</td>
<td>0.8</td>
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<tr>
<td>WV8A</td>
<td>A</td>
<td>200</td>
<td>1.06</td>
<td>0.010</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>45.8</td>
<td>61</td>
<td>0.8</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
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<td>196</td>
<td>1.06</td>
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<td>4.5</td>
<td>3</td>
<td>7</td>
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<td>55</td>
<td>1.0</td>
<td>3.45</td>
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</tbody>
</table>

Average = 1.4  
Std. Dev. = 0.51

$H_0$ Initial sample height  
$e_o$ Initial void ratio  
$\Delta e$ Change of void ratio at the end of test  
$V_e$ Applied voltage  
$K_h$ Coefficient of hydraulic conductivity  
$K_e$ Coefficient of electroosmotic permeability  
$\sigma_{os}$ Eq. (3), $x = 0.5H_e$  
$-U_{os}$ Magnitude of measured pore pressure, $x = 0.5H_e$  
$C_w$ Coefficient of electroosmotic consolidation
**Fig. 3** Typical Settlement Versus Time, Gloucester Clay

**Fig. 4** Typical Settlement Versus Time, Wallaceburg Clay
ELECTRO-Osmotic Consolidation Behavior of Two Ontario Clays

Electro-omotic Consolidation Pressure and Void Ratio

The results of mechanical consolidation of the Gloucester and Wallaceburg clays obtained from oedometer tests are presented as the solid lines in Fig. 5 and Fig. 6, respectively, with the results of electro-osmotic consolidation tests plotted on the same figures. The changes of void ratio were estimated from the volume change and settlement; while the computed increases of effective stresses and measured pore pressures are shown in columns (10-A) and (10-B) of Table 1, respectively. Figure 5 shows that the results of tests on the Gloucester clay samples yield reasonable agreement with the mechanical consolidation curve in both the recompression and virgin compression ranges. It should be noted that the excess pore pressure measurement was interrupted in some tests and those results were not presented here (Shang et al., 1996). Figure 6 presents the electroosmotic consolidation behavior of the Wallaceburg clay. The changes of the void ratio in the Wallaceburg clay samples were in the range of 0.01 to 0.04, about one order of magnitude lower than the Gloucester clay. As shown in Table 1 and Fig. 6, the electro-osmotic consolidation pressures generated during the tests on the Wallaceburg clay samples were less than 100 kPa, far below the preconsolidation pressure of the Wallaceburg clay (244 kPa). Therefore, consolidation in the virgin compression range did not occur. It is also seen from Fig. 6 that the measured and calculated values of electro-osmotic pore pressure yield much better agreement with each other. This could be attributed to the smaller settlement that reduced the interruption to the pore pressure probes. Note all tests on the Wallaceburg clay were conducted using cell A.

As a general remark, the results show that electro-osmotic consolidation of the Gloucester and Wallaceburg clays resembled the mechanical consolidation in terms of the relationship of void ratio and consolidation pressure. Furthermore, the magnitude of the negative electro-osmotic pore pressure was in general agreement with Eq. (3), provided that the effect of voltage loss between soil and electrode contacts was taken into account by an empirical factor. The settlement generated by electro-osmosis was shown to be dependent of the preconsolidation pressure of the soil. Therefore, electro-osmotic consolidation will be less effective in overconsolidated clays. The Wallaceburg clay illustrated the scenario. On the other hand, electro-osmotic consolidation can induce considerable settlement in normally consolidated clays, as shown in the case of the Gloucester clay.

Rate of Consolidation

The coefficients of electro-osmotic/mechanical consolidation versus consolidation pressure are plotted in Fig. 7 and Fig. 8 for the Gloucester and Wallaceburg clays, respectively. For the Gloucester clay, the ratio of $C_v$ in recompression to that in compression is about 20, which is characteristic of the sensitive clays encountered in eastern Canada (Terzaghi et al. 1996). The values of $C_w$ ranged from 0.65 m$^2$/a to 4.71 m$^2$/a, significantly lower than $C_v$, especially in the recompression range, as shown in Fig. 7. There appears to be an insignificant trend of decrease in $C_w$ values with the increase in the electro-osmotic consolidation pressure. The coefficient of mechanical consolidation of the Wallaceburg
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**Fig. 5 Consolidation Pressure Versus Void Ratio Change, Gloucester Clay**

**Fig. 6 Consolidation Pressure Versus Void Ratio Change, Wallaceburg Clay**
Fig. 7 Consolidation Pressure Versus Coefficient of Electro-osmotic/Mechanical Consolidation, Gloucester Clay

Fig. 8 Consolidation Pressure Versus Coefficient of Electro-osmotic/Mechanical Consolidation, Wallaceburg Clay
SHANG and HO

clay was 9.3 m$^2$/a in the recompression range (Lo and Becker, 1979). Figure 8 presents measured coefficients of electro-osmotic consolidation. It is shown that $C_v$ ranged from 0.8 m$^2$/a to 3.45 m$^2$/a, much lower than $C_e$ (9.3 m$^2$/a) and was not significantly affected by the consolidation pressure.

To summarize, the rates of electro-osmotic consolidation appeared to be lower than that of mechanical consolidation. Although more study is needed, electrochemical effects are considered to be the primary cause. Changes of Atterberg limits of the clays after the electro-osmotic consolidation tests on the two clays have been reported in Ho (1990) and Shang, et al. (1996), consistent with cases reported in the literature (as quoted in Mitchell, 1993). It was also observed that the increase of soil shear strength after electro-osmotic treatment could not be solely attributed to consolidation. Therefore, it was believed that electrochemical effects during an electro-osmotic process induce inter-particle bonding and cementation, thus resulting in a higher soil shear strength compared to mechanical consolidation. Another factor affecting the rate of electro-osmotic consolidation is the variation of excess pore pressure with time. As discussed before, the electro-osmotic permeability of the soil reduces with time, mainly due to formation of the dense, stiff crusts at electrode-soil contacts. Consequently, the ratio $k_v/k_e$ decreases, resulting in the decrease of the electro-osmotic pore pressure soon after it reached the maximum (Shang, et al., 1996). From an engineering design viewpoint, using $C_v$ obtained from mechanical consolidation tests in design could lead to underestimates of the required treatment time. Therefore, the electro-osmotic cell tests should be used whenever electro-osmotic consolidation is considered in a field application.

CONCLUSION

The electro-osmotic consolidation behavior of two soft clays from Ontario, Canada was investigated using two electro-osmotic testing cells. It was evident that electro-osmotic consolidation resembled the mechanical consolidation behavior of the soils. Taking the voltage loss between the electrode-soil contacts into account, reasonable agreement was found between the measured and computed negative electro-osmotic pore pressures. The coefficient of electro-osmotic consolidation of two clay soils was estimated. Although the results involved some uncertainty due to premature termination of some of the tests, it was found that the rate of electro-osmotic consolidation was considerably lower than the rate of mechanical consolidation. Therefore, the coefficient of electro-osmotic consolidation, $C_v$, determined from a laboratory electroosmotic test, should be used in the design of electro-osmotic consolidation instead of the conventional coefficient of consolidation obtained from consolidation (oedometer) tests.

ACKNOWLEDGMENT

The Natural Sciences and Engineering Research Council of Canada (NSERC) supported this study under Research Grant No. WFA0172832. Dr. K.Y. Lo, the Director of
ELECTRO-OSMOTIC CONSOLIDATION BEHAVIOR OF TWO ONTARIO CLAYS

the Geotechnical Research Centre at the University of Western Ontario, provided insightful
comments during the course of this research.

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Western Ontario, Canada.


APPROXIMATE CLOSED-FORM SOLUTION FOR A NON-LINEAR MODEL OF GEOSYNTHETIC-REINFORCED FOUNDATION SOILS

J.H. Yin

ABSTRACT

Most of one-dimensional (1-D) mathematical models for analyzing geosynthetic reinforcement of soft soils under footing loading are highly non-linear. Solutions are normally obtained using numerical techniques by iteration. This is a drawback of those simplified 1-D models. This paper presents a method for obtaining approximate closed-form solutions for a 1-D model (Yin 1997a). Results from the closed-form solution are compared to results from numerical finite difference calculations. The agreement is reasonable especially for small loading.

INTRODUCTION

Geosynthetics such as geomembranes and geogrids are often used to improve soft foundation soils. One of the techniques is to place engineered granular fill with the inclusion of a geosynthetic layer (geomembrane, geogrid or equivalent) on the soft foundation soils. A number of one-dimensional (1-D) models have been proposed for modeling this type of geosynthetic reinforcement for strip foundations (Bourdeau, et al., 1982; Love, et al., 1987; Madhav and Poorooshabh, 1988; Bourdeau, 1989; Poorooshabh, 1989; Poran, et al., 1989; Ghosh, 1991; Poorooshabh, 1991; Espinoza, 1994; Ghosh and Madhav, 1994; Khing, et al., 1994; Shukla and Chandra, 1994; Shukla and Chandra, 1995; Yin, 1997a; Yin, 1997b). Discussion on the limitation and applicability of this 1-D modeling approach can be found in Madhav and Poorooshabh (1988), Ghosh (1991), Ghosh and Madhav (1994), Shukla and Chandra (1994, 1995) and Yin (1997a).

This paper focuses on the approximate closed-form solution for a non-linear model of geosynthetic-reinforced foundation soils. All those 1-D model equations are highly non-linear and have to be solved using numerical techniques by iterations. This paper suggests a simple method to obtain an approximate closed-form solution for the non-linear model equations. A 1-D model proposed by Yin (1997a) is used to demonstrate the use and validation of the proposed method. The closed-form solution obtained can be used for the calculation of settlements and mobilized tension forces of the geosynthetic reinforcement under symmetric constant vertical pressure loading.

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MATHEMATICAL MODEL

Fig. 1(a) shows a simplified model for a strip footing on geosynthetic-reinforced soft soils. The soft soils are represented by Winkler springs. A geomembrane is included horizontally in engineered granular fills which are placed on the soft soils. The top and bottom fills are assumed to behave like Pasternak shear layers (Pasternak, 1954). The Pasternak shear layer assumes that the soil in the shear layer moves relatively to each other in vertical direction. Fig. 1(b) shows the top, bottom and the geosynthetic elements which are sheared in vertical direction.

Yin (1997a) suggested a deformation compatibility condition for the rough contact between (a) the top layer and the geomembrane and (b) the bottom layer and the geomembrane. Fig. 2(a) and Fig. 2(b) illustrate the deformation compatibility. Fig. 2(a) shows the shear deformation of the top and bottom elements due to the increase in geomembrane tension force $\Delta T$. Fig. 2(b) shows the stretching and rotation of the geomembrane element. The use of this compatibility condition eliminates the two uncertain constants ($\mu_t$ and $\mu_b$) for the top and bottom shear stresses acting on the geomembrane (Yin 1997a). And the tension stiffness modulus of the geomembrane is incorporated in the model (Yin 1997a). Using this compatibility condition, two non-linear differential equations are derived for solving two unknowns, that is, the settlement, $w$, and the tension force, $T$. The two equations (Yin 1997a) are

$$ q - k_s w = -(T + T_p) \cos \theta - \frac{d^2 w}{dx^2} \sin \theta \frac{dT}{dx} - \frac{d^2 w}{dx^2} (H_t G_t + H_b G_b) $$  

(1)

$$ \frac{d^2 T}{dx^2} = \sin \theta \cos \theta \frac{d^2 w}{dx^2} \frac{dT}{dx} + \frac{1}{H_t + G_t} \left[ \left( \frac{T}{H_t + G_t} \right)^2 + 1 - \left( \frac{dw}{dx} \right)^2 \right] $$  

(2)

where $q =$ pressure on the granular base; $k_s =$ spring constant; $w =$ settlement; $dx =$ projected element length in x-direction; $\theta =$ the rotation angle of element’s bottom side, related to $w$ by $\tan \theta = dw/dx$; $T =$ mobilized tension force; $T_p =$ pre-tension force; $H_t$ and $H_b$ are the thickness of the top and bottom fill layers, respectively; $G_t$ and $G_b$ are the average shear modulus of the top and bottom fill layers, respectively; $E_g =$ geomembrane stiffness modulus in kN/m. Eq. (1) and Eq. (2) are two non-linear ordinary differential equations. The non-linearity is due to large deflection (settlement) of the layers. It is noted in Eq. (1) that the pressure $q$ is an arbitrary pressure distribution on the granular base, including, for example, (a) the pressure within the footing and the zero pressure beyond the footing width as shown in Fig. 1(a).
Fig. 1 (a) Schematic Diagram of a 1-D Foundation Model and (b) Three Elements from a Vertical Segment of Infinitesimal Width, Forces and Stresses (Yin, 1997a)
Fig. 2 (a) Shear Deformations due to the Increase in Geomembrane Tension Force and (b) Stretching and Rotation of a Geomembrane Element (Yin 1997a)
APPROXIMATE CLOSED-FORM SOLUTION FOR A NON-LINEAR MODEL

The following normalization is used for Eq. (1) and Eq. (2)

\[ X = \frac{x}{B}, \quad W = \frac{w}{B}, \quad H_t^* = \frac{H_t}{B}, \quad H_b^* = \frac{H_b}{B} \]  

\[ G_t^* = \frac{G_t H_t}{k_t B^2}, \quad G_b^* = \frac{G_b H_b}{k_b B^2}, \quad E_g^* = \frac{E_g}{k_g B^2} \]  

\[ q^* = \frac{q}{k_g B}, \quad T_p^* = \frac{T_p}{k_p B^2}, \quad T^* = \frac{T}{k_i B^2} \]  

where \( B \) is half width of the footing pressure as shown in Fig. 1(a) and Fig. 3. All normalized items in Eq. (3) - Eq. (5) are dimensionless. The normalization makes Eq. (1) and Eq. (2) simpler.

Using the normalization in Eq. (3) - Eq. (5), Eq. (1) and Eq. (2) can be written as

\[ q^* = \sin \theta \frac{d^2T^*}{dx^2} \]  

\[ \frac{d^2T^*}{dx^2} = \sin \theta \cos \theta \frac{d^2W}{dx^2} + \frac{1}{\cos \theta} \left( \frac{G_t^*}{H_t^*} + \frac{G_b^*}{H_b^*} \right) \sqrt{\frac{T^*}{E_g^*} + 1 - \left( \frac{dW}{dx} \right)^2 - 1} \]  

where \( \tan \theta = dW/dx, \sin \theta = (dW/dx)/\sqrt{1 + (dW/dx)^2} \), and \( \cos \theta = 1/\sqrt{1 + (dW/dx)^2} \).

For the geosynthetic reinforcement system in Fig. 3, four boundary conditions are: at \( X = 0 \) (or \( x = 0 \)), due to symmetry, the slope, \( dW/dx \), shall be zero and the rate of tension force increase, \( dT^*/dx \), shall be zero (Yin 1997a, 1997b), that is:

\[ \frac{dW}{dx} = 0 \]  

\[ \frac{dT^*}{dx} = 0 \]  

At \( X = L^* = L/B \) (or \( x = L \)), the right end of the geomembrane is free, so that the mobilized tension force \( T \) shall be zero. The shear stress on the right vertical side (at
Fig. 3 A Symmetric Geomembrane Reinforced Foundation System

\[ X = L^* = L/B \] shall be zero. Since the Pasternak shear layer assumes that \( \tau = G_e \frac{dw}{dx} \) and \( \tau^* = G_b \frac{dw}{dx} \), so that \( \frac{dw}{dx} = 0 \). Thus the boundary conditions at \( X = L^* \) are (Yin 1997a, 1997b)

\[
\frac{dW}{dX} = 0
\]  \hspace{1cm} (10)

\[
\tau^* = 0
\]  \hspace{1cm} (11)

Eq. (6) and Eq. (7) are two non-linear ordinary differential equations which can be solved to obtain a unique solution for given boundary conditions (Yin 1997a). Since the equations are highly non-linear, numerical techniques and iteration computing procedures must be used to obtain a solution using a computer program. Yin (1997a) found that the convergence using a finite difference method was slow. This will limit the engineering applications of the 1-D model (Yin 1997a).

**APPROXIMATE SOLUTION**

This paper suggests a simple method to obtain an approximate closed-form solution for those non-linear 1-D model equations. This method entails that the non-linear items in the model equations are approximately replaced by an average value of corresponding variables. This paper uses Yin's 1-D model as in Eq. (6) and Eq. (7) to demonstrate the use and validation of the proposed approximate solution method.
APPROXIMATE CLOSED-FORM SOLUTION FOR A NON-LINEAR MODEL

Using the average value of rotation angle, \( \bar{\theta} \), tension force, \( T^* \), the rate of tension force increase, \( (dT^*/dX)_{avg} \), Eq. (6) becomes:

\[
q^* = W - \sin \bar{\theta} \left( \frac{dT^*}{dX} \right)_{avg} - \left[ \frac{1}{2} \left( T^* + T^* \right) \cos^3 \bar{\theta} + (G^* + G^*) \right] \frac{d^2W}{dX^2}
\]  

(12)

The item \( (\sin \theta \cos \theta)(d^2W/dX^2)(dT^*/dX) \) in Eq. (7) is a small value (<1) of higher order for relatively small deflection (settlement) compared to other items. This item is ignored as an approximate approach. Thus, using average \( \tan \bar{\theta} \), \( \cos \bar{\theta} \), and \( T^* \), Eq. (7) becomes:

\[
\frac{d^2T^*}{dX^2} = -f
\]

(13)

where

\[
f = \frac{1}{\cos \bar{\theta}} \left( \frac{G^*}{H_t^*} + \frac{G^*}{H_b^*} \right) \left[ 1 - \sqrt{\left( \frac{T^*}{E_g^*} \right)^2 + 1 - (\tan \bar{\theta})^2} \right] > 0
\]

where \( f \) is considered constant approximately when integrating Eq. (13). In the expression for \( f \), the value of \( [(T^*/E_g^*)^2 + 1 - (\tan \bar{\theta} )^2] \) shall be larger than or equal to zero. Normally \( [(T^*/E_g^*)^2 + 1 - (\tan \bar{\theta} )^2] \) is smaller than 1. Thus the value of \( f \) shall be positive.

Referring to Fig. 3, those average values can be calculated in the following expressions:

\[
\sin \bar{\theta} = -\frac{W_o - W_L^*}{\sqrt{(W_o - W_L^*)^2 + L^*^2}} < 0
\]

(14)

\[
\cos \bar{\theta} = \frac{L^*}{\sqrt{(W_o - W_L^*)^2 + L^*^2}}
\]

(15)

\[
\tan \bar{\theta} = -\frac{W_o - W_L^*}{L^*} < 0
\]

(16)
\[
\bar{T}^* = \frac{T_L^* + T_0^*}{2}
\]

(17)

\[
\left(\frac{dT^*}{dX}\right)_{avg} = \frac{T_L^* - T_0^*}{L^*}
\]

(18)

where \(W_0\) and \(W_L\) are the settlements at \(X = 0\) and \(X = L^*\); and \(T_0^*\) and \(T_L^*\) are the mobilized tension forces at \(X = 0\) and \(X = L^*\).

Integrating Eq. (13) twice leads to

\[
T^* = -\frac{1}{2} f X^2 + c_1 X + c_2
\]

(19)

where \(c_1\) and \(c_2\) are two arbitrary constants. Using the two boundaries, that is, \(dT^*/dX = 0\) at \(X = 0\) and \(T^* = 0\) at \(X = L^* = L/B\), the two constants in Eq. (19) are determined. And Eq. (19) becomes:

\[
T^* = \frac{1}{2} f (L^*^2 - X^2)
\]

(20)

Since \(f > 0\), \(T^*\) in Eq. (20) shall be positive (tension) and vary from a maximum of \(T^* = T_0^* = \frac{1}{2} f L^*^2\) at \(X = 0\) to \(T^* = T_L^* = 0\) at \(X = L^*\) for a symmetric problem as shown in Fig. 3. The average tension force is:

\[
\bar{T}^* = \frac{T_L^* + T_0^*}{2} = \frac{1}{4} f L^*^2
\]

(21)

The average tension increase rate is:

\[
\left(\frac{dT^*}{dX}\right)_{avg} = \frac{T_L^* - T_0^*}{L^*} = -\frac{1}{2} f L^*
\]

(22)

In Eq. (12), using \(TG\) to denote \([(\bar{T}^* + T_p^*) \cos \bar{\theta} + (G_t^* + G_b^*)\), Eq. (12) becomes:

\[
TG\frac{d^2W}{dX^2} - W = -q^* \sin \bar{\theta} \left(\frac{dT^*}{dX}\right)_{avg}
\]

(23)
APPROXIMATE CLOSED-FORM SOLUTION FOR A NON-LINEAR MODEL

Referring to Fig. 3, the constant pressure $q^* = q_o^*$ for $0 \leq X \leq 1$ and $q^* = 0$ for $1 \leq X \leq L^*$ can be expressed in a Fourier series (Kreyszig, 1993) as:

$$q^* = \frac{q_o^*}{L^*} + 2q_o^* \sum_{n=1}^{\infty} \frac{n\pi}{L^*} \frac{\sin{n\pi X}}{L^*} \frac{\cos{n\pi X}}{L^*}$$

(24)

The solution to Eq. (23) using Eq. (24) is:

$$W = c_3 \exp\left(\frac{X}{\sqrt{TG}}\right) + c_4 \exp\left(-\frac{X}{\sqrt{TG}}\right) + \sin\bar{\theta} \left(\frac{dT^*}{dX}\right)_{avg} +$$

$$\frac{q_o^*}{L^*} + 2q_o^* \sum_{n=1}^{\infty} \frac{n\pi}{L^*} \frac{1}{\sqrt{TG}} \frac{\cos{n\pi X}}{L^*}$$

(25)

where $c_3$ and $c_4$ are two arbitrary constants. Using the boundary conditions: $X = 0$, $dW/dX = 0$ and $X = L^*$, $dW/dX = 0$, $c_3$ and $c_4$ are found to be zero and the solution in Eq. (25) becomes

$$W = \sin\bar{\theta} \left(\frac{dT^*}{dX}\right)_{avg} + \frac{q_o^*}{L^*} + 2q_o^* \sum_{n=1}^{\infty} \frac{\cos{n\pi X}}{L^*}$$

(26)

From Eq. (26), at $X = 0$, $W_o$ is:

$$W_o = \sin\bar{\theta} \left(\frac{dT^*}{dX}\right)_{avg} + \frac{q_o^*}{L^*} + 2q_o^* \sum_{n=1}^{\infty} \frac{\cos{n\pi X}}{L^*}$$

(27)

at $X = L^*$, $W_L$ is:

$$W_L = \sin\bar{\theta} \left(\frac{dT^*}{dX}\right)_{avg} + \frac{q_o^*}{L^*} + 2q_o^* \sum_{n=1}^{\infty} \frac{\cos{n\pi X}(-1)^n}{L^*}$$

(28)
The \( W_o \) and \( W_L \) from Eq. (27) and Eq. (28) can be used in Eq. (15) and Eq. (16) to calculate the average value of \( \tan \theta \) and \( \cos \theta \).

Eq. (20) and Eq. (26) are two approximate equations for solving \( T^* \) and \( W^* \). To calculate \( T^* \) and \( W^* \), initial values (say zero) of \( W_L, W_o \) and \( T_o^* \) are assumed and used in Eq. (20) and Eq. (28). A few iterations are carried out to make \( W_L^{(k+1)} \approx W_L^{(k)}, W_o^{(k+1)} \approx W_o^{(k)} \), and \( T_o^*^{(k+1)} \approx T_o^*^{(k)} \), where \( k \) is iteration index. It is found that the convergence can be achieved quickly for \( k < 10 \). The number of items in Eq. (26), Eq. (27) and Eq. (28) can be taken as \( n = 10 \) or smaller.

RESULTS, COMPARISON AND DISCUSSION

The non-linear Eq. (6) and Eq. (7) have been solved using a finite difference technique by iteration (Yin 1997a). Results from the finite difference solution are compared to results from a finite element model and three 1-D models by Shukla and Chandra (1995), Ghosh (1991), and Madhav and Poorooshab (1988). It is found that the results from Yin's 1-D model are in good agreement with the finite element model results (Yin 1997a).

Fig. 4 shows the comparison of the results using the approximate closed-form solution presented above and the results from the finite difference (FD) solution (Yin 1997a) for \( q^* = 0.1, 0.3 \) and \( 0.8 \). The normalized pressure \( q^* \) is defined in Eq. (3). Other parameters are shown in Fig. 4. It is seen from Fig. 4 that the predicted settlements from the closed-form solution are very close to the settlements from the FD solution for \( q^* = 0.1 \) and \( q^* = 0.3 \). For \( q^* = 0.8 \), the settlement from the closed-form solution has an error ranging from 0% to 10%. The mobilized tension force from the closed-form solution is in good agreement with the FD solution for \( q^* = 0.1 \). But the error is getting bigger for larger \( q^* \)-values. The error range is from 20% to 25%.

The settlement and tension force at the symmetric line \( (X = 0) \) are maximum, that is \( W_{max} = W_o \) and \( T_{max}^* = T_o^* \). The maximum settlement and tension force are normally the controlling factor for design purposes. It is seen from Fig. 4 that the error range for the calculation of the maximum settlement and tension force is from 0% to 10% for loading \( q^* \) up to 0.8. The obtained approximate closed-form solution can be easily carried out using any spreadsheet program.

CONCLUSIONS

Most mathematical models for geosynthetic reinforcement of soft soils are a set of non-linear differential equations which are difficult to be solved. This paper suggests a simple method by using average values in the model equations to obtain an approximate closed-form solution to the non-linear problems. By comparing to the
Fig. 4 Comparison to Finite Difference Solution; (a) Settlements and (b) Mobilized Tension Forces for Different Loading $q^*$
results from rigorous finite difference solutions, the approximate closed-form solution generally gives accurate settlements and tension forces for small loading. The error is getting bigger for larger loading with a relative error of 25% for loading $q^*$ up to 0.8. However, the error for the maximum settlement and tension force is smaller, only 10% for $q^*$ up to 0.8 for the problem studied in this paper. The calculation using the proposed Fourier series solution can be easily carried out using a spreadsheet program. The suggested solution method can be applied for obtaining approximate solutions for other non-linear 1-D models. The solution obtained may be easy to use for design analyses.

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APPROXIMATE CLOSED-FORM SOLUTION FOR A NON-LINEAR MODEL


CONSOLIDATION OF GROUND WITH PARTIALLY PENETRATED VERTICAL DRAINS

X.W. Tang\textsuperscript{1} and K. Onitsuka\textsuperscript{2}

ABSTRACT

Due to the complex nature of the problem, the consolidation of ground with partially penetrated vertical drains in the past are analyzed only by numerical method. For practical application, some simple approximate methods have been developed. A semi-analytical solution for consolidation of ground with partially penetrated vertical drains is presented in this paper. A simple calculation program is developed for the same. The existing approximate methods are discussed. The influences of well resistance and penetration fraction on the consolidation of ground with partially penetrated vertical drains are shown in the figures.

INTRODUCTION

Vertical drains are widely used to accelerate the consolidation process of soft clayey foundation and to improve the strength of soft clay. Usually vertical drains are installed to fully penetrate the soft clay layer. Many analytical solutions for consolidation of homogeneous soft soil by vertical drains, namely single-layered with fully penetrated vertical drains, were developed (Barron, 1948; Yoshikuni and Nakanoda, 1974; Hansbo, 1981; Onoue, 1988b; Zeng and Xie, 1989; Tang and Onitsuka, 1997a). However, in some instances, the soft clay is so deep that it is not economical to penetrate to full depth, or surcharge loading is too small to justify full penetration of soft clay layer. Due to the complexity of the problem, the consolidation of ground with partially penetrated vertical drains, is analyzed only by numerical methods (Ruesson, et al., 1985; Onoue, 1988a; Nakano and Okuie, 1991). Some simple approximate methods have been developed for easier application (Hart, et al., 1958; Zeng and Xie, 1989).

A semi-analytical solution of the consolidation problem of double-layered ground with fully penetrated vertical drains has been obtained by Tang and Onitsuka (1997b). When the system of ground with partially penetrated vertical drains is separated into two parts from the bottom of vertical drains, the upper part is considered as the single-layered ground with fully penetrated vertical drains whereas the lower part is considered as the simple single-layered ground. Similar to double-layered ground with fully penetrated vertical drains, the consolidation problem of ground with partially penetrated vertical drains may also be solved using the same procedure.

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A semi-analytical solution of consolidation of ground with partially penetrated vertical drains is presented in this paper. The main steps of computation procedure are listed. The existing approximate methods are discussed, and some characteristics of consolidation of ground with partially penetrated vertical drains are analyzed.

**BASIC ASSUMPTIONS AND MATHEMATICAL MODELLING**

For easier discussion, similar to Hart, et al. (1958) and Runesson, et al. (1985), the horizontal plane through the bottom of the vertical drains is called the plane of penetration. The section above the plane of penetration is defined as the section with vertical drains and the section below the plane of penetration is defined as the section without vertical drains.

The basic assumptions in this study are listed as follows:

1. The section with vertical drains satisfies the consolidation of the homogeneous ground with vertical drains under quasi-equal strain condition,
   
   a) Quasi-equal strain hypothesis is valid, namely, the horizontal sections remain horizontal and the boundary of a cylindrical body has no horizontal displacement. The vertical part of consolidation of the soil considers the average excess pore water pressure of the soil at any depth.
   
   b) The total inflow of pore water through the boundary of the vertical drains is equal to the upper flow of water into the vertical drains.
   
   c) The coefficient of volume compressibility of the smear zone is the same as that of the natural soil.
   
   d) Radial water flow in the vertical drains is neglected.

2. The section without vertical drains satisfies the assumptions of Terzaghi's one dimensional consolidation.

3. The continuity conditions at the plane of penetration are:

   a) The excess pore water pressure within the vertical drains is equal to the excess pore water pressure of the section without vertical drains.

   b) The average excess pore water pressure of soil of the section with vertical drains, is equal to the excess pore water pressure of the section without vertical drains.

   c) The total water flow of the vertical drains and the soil zone of the section with vertical drains is equal to the water flow of the section without vertical drains.
CONSOLIDATION OF GROUND

Assumptions (1) and (2) are applicable for a short distance away from the plane of penetration. Although the continuity conditions near the plane of penetration are very complex, from the viewpoint of balance of total quantity of water flow and, excess pore water pressure, assumption (3) is made.

For definition of consolidation of ground with partially penetrated vertical drains, the natural soil of the section with vertical drains and the soil of the section without vertical drains are homogeneous. For general application, the mechanical parameters of the natural soil of the section with vertical drains and the soil of the section without vertical drains are made different.

The analysis scheme is shown in Fig. 1. According to the above assumptions, the simultaneous basic equations of the system are:

- For the section with vertical drains, $0 < z < h_f$, the simultaneous basic equations are:

  \begin{align*}
  &\frac{k_{s1}}{m_{s1} \gamma_w} \left( \frac{1}{r} \frac{\partial u_{s1}}{\partial r} + \frac{\partial^2 u_{s1}}{\partial r^2} \right) + \frac{k_{s1}}{m_{s1} \gamma_w} \frac{\partial^2 \bar{u}_{s1}}{\partial z^2} = \frac{\partial \bar{u}_{s1}}{\partial t}, \quad r_w \leq r \leq r_s 
  \end{align*}

  \text{where: } h_f = \text{length of vertical drain}; \ r_w = \text{radius of vertical drain}; \ r_s = \text{radius of smear zone}; \ r = \text{radial coordinate}; \ z = \text{vertical coordinate}; \ t = \text{time}; \ m_{s1} = \text{coefficient of volume compressibility of the soil of the section with vertical drains}; \ k_{s1} = \text{vertical coefficient of permeability of the soil of the section with vertical drains}; \ k_{s1} = \text{horizontal coefficient of permeability of remolded soil}; \ \gamma_w = \text{unit weight of water}; \ u_{s1}(r, z, t) = \text{excess pore water pressure at any point in the smear zone}; \ \bar{u}_{s1}(z, t) = \text{average excess pore water pressure of the section with vertical drains at any depth}.

- Natural soil,

  \begin{align*}
  &\frac{k_{h1}}{m_{h1} \gamma_w} \left( \frac{1}{r} \frac{\partial u_{h1}}{\partial r} + \frac{\partial^2 u_{h1}}{\partial r^2} \right) + \frac{k_{h1}}{m_{h1} \gamma_w} \frac{\partial^2 \bar{u}_{h1}}{\partial z^2} = \frac{\partial \bar{u}_{h1}}{\partial t}, \quad r_s \leq r \leq r_s 
  \end{align*}

  \text{where: } r_s = \text{radius of the vertical drain influence zone}; \ u_{h1}(r, z, t) = \text{excess pore water pressure at any point in the natural soil zone of the section with vertical drains}; \ k_{h1} = \text{horizontal coefficient of permeability of the natural soil of the section with vertical drains}.

Continuity at vertical drain cylindrical surface,

\begin{align*}
  \frac{\partial^2 u_w}{\partial z^2} = -2 \frac{k_{s1}}{r_w k_w} \left( \frac{\partial u_{s1}}{\partial r} \right) \bigg|_{r=r_w}
\end{align*}

\[211\]
where: $u_w (z, t)$ = excess pore water pressure within the vertical drains; $k_w$ = coefficient of permeability of the vertical drains.

Average excess pore water pressure on horizontal plane,

$$
\bar{u}_1 = \frac{1}{\pi (r^2 - r_s^2)} \left( \int_{r_s}^{r} 2 \pi r u_{w1} \, dr + \int_{r_s}^{r} 2 \pi r u_{n1} \, dr \right)
$$

The corresponding boundary conditions are:

Impermeable wall at $r = r_s$,

$$
\frac{\partial u_{n1}}{\partial r} = 0
$$

Continuity of pore pressure at $r = r_w$,

$$
u_{s1} = u_w
$$

Continuity of pore pressure at $r = r_s$,

$$
u_{s1} = u_{n1}
$$

Continuity of pore pressure gradient at $r = r_s$,

$$
k_{s1} \frac{\partial u_{s1}}{\partial r} = k_{h1} \frac{\partial u_{n1}}{\partial r}
$$
CONSOLIDATION OF GROUND

At top of clay layer, $z = 0$, 

$$u_w = 0 \quad (9)$$

and, 

$$\bar{u}_1 = 0 \quad (10)$$

- For the section without vertical drains, $h_1 \leq z \leq H$, the basic equation is:

$$\frac{k_{oz}}{m_{oz}\gamma} \frac{\partial^2 u_2}{\partial z^2} = \frac{\partial u_2}{\partial t} \quad (11)$$

where: $H =$ whole thickness of soil; $m_{oz} = \text{coefficient of volume compressibility of the soil of the section without vertical drains}; k_{oz} = \text{vertical coefficient of permeability of the soil of the section without vertical drains}; u_2 (z, t) =$ excess pore water pressure of the section without vertical drains at any depth.

The corresponding boundary condition is:

For impervious bottom (PTIB) $z = H$, 

$$\frac{\partial u_2}{\partial z} = 0 \quad (12)$$

or,

For pervious bottom (PTPB) $z = H \quad u_2 = 0 \quad (13)$

where: PTIB = drainage condition with Pervious Top and Impervious Bottom; PTPB = drainage condition with Pervious Top and Pervious Bottom.

The continuity conditions at the plane of penetration, $z = h_1$, are:

$$u_w = u_2 \quad (14)$$

$$\bar{u}_1 = u_2 \quad (15)$$

$$k_{oz} \pi (r_e^2 - r_w^2) \frac{\partial u_1}{\partial z} + k_w \pi r_w^2 \frac{\partial u_w}{\partial z} = k_{oz} \pi r_e^2 \frac{\partial u_2}{\partial z} \quad (16)$$

The initial condition of the system is:

$$t = 0 \quad \bar{u}_1(z) = u_0(z) = q_0 \quad u_2(z) = u_0(z) = q_0 \quad (17)$$

where: $u_0 (z) =$ initial pore pressure; $q_0 =$ constant loading.
SOLUTION OF SYSTEM

Solution for Section with Vertical Drains

From the Appendix, the solutions for \( u_w \) and \( \tilde{u}_1 \) can be expressed as:

\[
\begin{align*}
\ u_w &= \sum_{m=0}^{\infty} A_m \left[ a_m \sin \left( \lambda_m \frac{z}{H} \right) + c_m \sinh \left( \xi_m \frac{z}{H} \right) \right] e^{-\beta_m t} \\
\ 
\tilde{u}_1 &= \sum_{m=0}^{\infty} A_m \left[ a_m \left( 1 + \frac{1}{\varphi_1} \lambda_m^2 \right) \sin \left( \lambda_m \frac{z}{H} \right) + c_m \left( 1 - \frac{1}{\varphi_1} \xi_m^2 \right) \sinh \left( \xi_m \frac{z}{H} \right) \right] e^{-\beta_m t}
\end{align*}
\]

(18)  (19)

where: \( a_m, c_m \) and \( A_m \) are arbitrary constants. \( a_m \) is included in \( A_m \). Hence, \( A_m = 1 \).

\[
\begin{align*}
\varphi_1 &= (n^2 - 1) \left( \frac{2}{F} \frac{k_{hl}}{k_w} \right)^2, \quad s = \frac{r_s}{r_w}, \quad n = \frac{r_s}{r_w}, \\
F &= \left( \ln \frac{n}{s} + \frac{k_{hl}}{k_s} \ln \frac{3}{4} \frac{s^2}{n^2 - 1} \right) \frac{n^2}{n^2 - 1} + \frac{s^2}{n^2 - 1} \left( 1 - \frac{k_{hl}}{k_s} \right) \left( 1 - \frac{s^2}{4n^2} \right) + \frac{k_{hl}}{k_s} \left( 1 - \frac{1}{4n^2} \right) \\
&= \left( \ln \frac{n}{s} + \frac{k_{hl}}{k_s} \ln \frac{3}{4} \frac{s^2}{n^2 - 1} \right) \frac{n^2}{n^2 - 1} + \frac{s^2}{n^2 - 1} \left( 1 - \frac{k_{hl}}{k_s} \right) \left( 1 - \frac{s^2}{4n^2} \right) + \frac{k_{hl}}{k_s} \left( 1 - \frac{1}{4n^2} \right)
\end{align*}
\]

Solution for Section without Vertical Drains

For the Impervious Bottom condition (PTIB), from the Appendix, the solution for \( u_2 \) can be expressed as:

\[
\begin{align*}
\ u_2 &= \sum_{m=0}^{\infty} A_m b_{m2} \cos \left( \lambda_m \frac{z}{H} \right) e^{-\beta_m t} 
\end{align*}
\]

(20)

Solution of Total System

The general solutions for the section with vertical drains and the section without vertical drains have been independently obtained. The next step is to solve the total system by virtue of the continuity conditions at the plane of penetration \( (z = h) \), and the orthogonal relation of the system.

For any value of \( t \), \( \tilde{u}_1(h, t) = u_2(h, t) \), must be found, hence:

\[
\beta_{m1} = \beta_{m2} = \beta_m
\]

(21)

Substituting Eq. (18) \sim Eq. (21) into the continuity conditions at the plane of penetration, Eq. (14) \sim Eq. (16), and changing to matrix form.
CONSOLIDATION OF GROUND

\[ S X^T = 0 \]  \hfill (22)

where:

\[
S = \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\]  \hfill (23)

\[ X = [a_{m1} \quad c_{m1} \quad b_{m2}] \]  \hfill (24)

and,

\[ S_{11} = \sin(\lambda_{m1} \rho) \]

\[ S_{12} = \sinh(\xi_{m1} \rho) \]

\[ S_{13} = -\cos[\lambda_{m2} (1 - \rho)] \]

\[ S_{21} = \left(1 + \frac{1}{\varphi_{1}} \lambda_{m1}^2\right) \sin(\lambda_{m1} \rho) \]

\[ S_{22} = \left(1 - \frac{1}{\varphi_{1}} \xi_{m1}^2\right) \sinh(\xi_{m1} \rho) \]

\[ S_{23} = -\cos[\lambda_{m2} (1 - \rho)] \]

\[ S_{31} = \left[\frac{1}{n^2} k_w + \frac{n^2 - 1}{n^2} \left(1 + \frac{1}{\varphi_{1}} \lambda_{m1}^2\right) k_{v1}\right] \lambda_{m1} \cos(\lambda_{m1} \rho) \]

\[ S_{32} = \left[\frac{1}{n^2} k_w + \frac{n^2 - 1}{n^2} \left(1 - \frac{1}{\varphi_{1}} \xi_{m1}^2\right) k_{v1}\right] \xi_{m1} \cosh(\xi_{m1} \rho) \]

\[ S_{33} = -k_{v2} \lambda_{m2} \sin[\lambda_{m2} (1 - \rho)] \].
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where, \( \rho = h/DD, \) penetration fraction.

In order to get unequal zero solutions of \( X \) order:

\[
\det S = 0
\]

(25)

The value of \( \beta_m \) can be obtained from the above equation. The number of \( \beta_m \) is infinite and the term \( m \) is only the serial number. Substituting for every value of \( \beta_m \) in Eq. (22), and ordering, \( a_{m1} = 1 \), the corresponding values of \( c_{m1} \) and \( b_{m2} \) are obtained.

The last problem is to determine the value of \( A_m \).

The procedure for proving the orthogonal relation of the system, is similar to the procedure for proving the orthogonal relation of double-layered ground with fully penetrated vertical drains (Tang and Onitsuka, 1997b).

The functions of \( Z_{m1}(z) \) and \( Z_{m2}(z) \) satisfy the following orthogonal relation:

\[
(n^2 - 1)m_{12} \int_0^H Z_{m1}Z_{m1}dz + n^2 m_{22} \int_0^H Z_{m2}Z_{m2}dz = 0 \quad m \neq m'
\]

(26)

where \( Z_{m1}(z) \) and \( Z_{m2}(z) \) are some functions of \( z \) as shown in the Appendix.

According to the above equation for the orthogonal relation of the system and the initial condition of the system, the value of \( A_m \) can be determined as:

\[
A_m = \frac{(n^2 - 1)m_{12} \int_0^H u_0(z)g_{m1}(z)dz + n^2 m_{22} \int_0^H u_0(z)g_{m2}(z)dz}{(n^2 - 1)m_{12} \int_0^H g_{m1}^2(z)dz + n^2 m_{22} \int_0^H g_{m2}^2(z)dz}
\]

(27)

If the initial excess pore water pressure is caused by loading only, and does not relate to the depth, the above equation can be simplified as:

\[
A_m = \frac{(n^2 - 1)m_{12}u_0 \int_0^H g_{m1}(z)dz + n^2 m_{22}u_0 \int_0^H g_{m2}(z)dz}{(n^2 - 1)m_{12} \int_0^H g_{m1}^2(z)dz + n^2 m_{22} \int_0^H g_{m2}^2(z)dz} = \frac{(n^2 - 1)m_{12}W_{m1} + n^2 m_{22}u_0 W_{m2}}{(n^2 - 1)m_{12}W_m + n^2 m_{22} W_{m4}}
\]

(28)

where:

\[
W_{m1} = \int_0^H g_{m1}(z)dz
\]

\[
= a_{m1} \left( 1 + \frac{1}{\varphi_1} \right) \frac{H}{\lambda_{m1}} \left[ 1 - \cos(\lambda_{m1}\rho) \right] + c_{m1} \left( 1 - \frac{1}{\varphi_1} \right) \frac{H}{\varphi_1} \left[ \cosh(\varphi_1\rho) - 1 \right]
\]
\[ W_{m2} = \int_0^H g_{m2}(z) \, dz = b_{m2} \frac{H}{\lambda_{m2}} \sin[\lambda_{m2}(1 - \rho)] \]

\[ W_{m3} = \int_0^h g_{m3}^2(z) \, dz \]

\[ = \frac{1}{2} a_{m1}^2 \left(1 + \frac{1}{\varphi_1} \lambda_{m1}^2 \right)^2 \left[h_1 - \frac{H}{2\lambda_{m1}} \sin(2\lambda_{m1}\rho) \right] \]

\[ + \frac{1}{2} e_{m1}^2 \left(1 - \frac{1}{\varphi_1} \xi_{m1}^2 \right)^2 \left[ \frac{H}{2\xi_{m1}} \sinh(2\xi_{m1}\rho) - h_1 \right] \]

\[ + 2a_{m1}c_{m1} \left(1 + \frac{1}{\varphi_1} \lambda_{m1}^2 \right) \left(1 - \frac{1}{\varphi_1} \xi_{m1}^2 \right) \frac{1}{\lambda_{m1}^2 + \xi_{m1}^2} \]

\[ \times \left[ \xi_{m1} H \sin(\lambda_{m1}\rho) \cosh(\xi_{m1}\rho) - \lambda_{m1} H \cos(\lambda_{m1}\rho) \sinh(\xi_{m1}\rho) \right] \]

\[ W_{m4} = \int_0^H g_{m4}^2(z) \, dz = \frac{1}{2} b_{m2}^2 \left[ (H - h_1) + \frac{H}{2\lambda_{m2}} \sin[2\lambda_{m2}(1 - \rho)] \right] \]

The average degree of consolidation at any depth is:

\[ U_1(z) = 1 - \frac{u_1}{u_0} = 1 - \frac{1}{u_0} \sum_{m=0}^\infty A_m g_{m1}(z)e^{-\beta m} \quad (29) \]

\[ U_2(z) = 1 - \frac{u_2}{u_0} = 1 - \frac{1}{u_0} \sum_{m=0}^\infty A_m g_{m2}(z)e^{-\beta m} \quad (30) \]
The average degree of consolidation for the section with vertical drains and the section without vertical drains are:

$$
\bar{U}_1 = 1 - \frac{1}{h_1} \int_0^h \frac{-u_1}{u_0} \, dz = \int_0^h U_1 \, dz = 1 - \frac{1}{u_0} \sum_{m=0}^\infty A_m W_m e^{-\lambda_m t} 
$$

(31)

$$
\bar{U}_2 = 1 - \frac{1}{h_2} \int_h^H \frac{-u_2}{u_0} \, dz = \int_h^H U_2 \, dz = 1 - \frac{1}{u_0} \sum_{m=0}^\infty A_m W_m e^{-\lambda_m t} 
$$

(32)

The overall average degree of consolidation for whole thickness of soil defined by the excess pore water pressure is:

$$
\bar{U}_p = 1 - \frac{1}{u_0 H} \left( \int_0^h \frac{-u_1}{u_0} \, dz + \int_h^H \frac{-u_2}{u_0} \, dz \right) = 1 - \frac{1}{u_0 H} \sum_{m=0}^\infty A_m (W_{m1} + W_{m2}) e^{-\lambda_m t} 
$$

(33)

The overall average degree of consolidation for whole thickness of soil defined by the total settlement is:

$$
\bar{U}_s = 1 - \frac{m_1 \int_0^h \frac{-u_1}{u_0} \, dz + m_2 \int_h^H \frac{-u_2}{u_0} \, dz}{u_0 (m_1 h_1 + m_2 h_2)} = 1 - \frac{\sum_{m=0}^\infty A_m (m_1 W_{m1} + m_2 W_{m2}) e^{-\lambda_m t}}{u_0 (h_1 m_1 + h_2 m_2)} 
$$

(34)

It can be seen that $\bar{U}_p \neq \bar{U}_s$ except for the condition $m_{1I} = m_{2I}$. For consistent definition of consolidation of ground with partially penetrated vertical drains, the mechanical parameters of the natural soil of the section with vertical drains, are equal to those of the soil of the section without vertical drains. Then, $m_{1I} = m_{2I}$, hence, $\bar{U}_p = \bar{U}_s = \bar{U}$, where $\bar{U}$ is defined as the overall average degree of consolidation for whole thickness of soil, for the consistent definition of consolidation of ground with partially penetrated vertical drains.

Solution for PTPB (Pervious Bottom)

The simultaneous basic partial differential equations of the system and the solution conditions for PTPB are the same as those for PTIB, except for the boundary condition at $Z = H$, $u_2 = 0$.

For the first layer, the general solutions for $u_w$ and $\bar{u}_1$ are the same as those for PTIB:

$$
u_w = \sum_{m=0}^\infty A_m \left[ a_m \sin \left( \lambda_m \frac{z}{H} \right) + c_m \sinh \left( \xi_m \frac{z}{H} \right) \right] e^{-\lambda_m t}
$$

(35)
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\[ \bar{u}_1 = \sum_{m=0}^{\infty} A_m \left[ a_m \left( 1 + \frac{1}{\varphi_1} \lambda_m^2 \right) \sin \left( \lambda_m \frac{z}{H} \right) + c_m \left( 1 - \frac{1}{\varphi_1} \xi_m^2 \right) \sinh \left( \xi_m \frac{z}{H} \right) \right] e^{-\rho_m t} \]  \hspace{1cm} (36)

From the Appendix, the general solution for \( u_2 \) is:

\[ u_2 = \sum_{m=0}^{\infty} A_m a_{m2} \sin \left( \lambda_m \left( 1 - \frac{z}{H} \right) \right) e^{-\rho_m t} \]  \hspace{1cm} (37)

By virtue of the continuity conditions at the plane of penetration, Eq. (14) ~ Eq. (16), the following matrix equation is obtained:

\[ S \times X^T = 0 \]  \hspace{1cm} (38)

where:

\[ S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \]  \hspace{1cm} (39)

\[ X = \begin{bmatrix} a_{m1} & a_{m2} & c_{m1} \end{bmatrix} \]  \hspace{1cm} (40)

and:

\[ S_{11} = \sin(\lambda_{m1}\rho) \]

\[ S_{12} = \sinh(\xi_{m1}\rho) \]

\[ S_{13} = -\sin[\lambda_{m2}(1-\rho)] \]

\[ S_{21} = \left( 1 + \frac{1}{\varphi_1} \lambda_{m1}^2 \right) \sin(\lambda_{m1}\rho) \]

\[ S_{22} = \left( 1 - \frac{1}{\varphi_1} \xi_{m1}^2 \right) \sinh(\xi_{m1}\rho) \]

\[ S_{31} = -\sin[\lambda_{m2}(1-\rho)] \]
\[ s_{\nu} = \left[ \frac{1}{n^2} k_{\nu} + \frac{n^2 - 1}{n^2} k_{\nu} \left( 1 + \frac{1}{\varphi_1} \lambda_{m1}^2 \right) \right] \lambda_{m1} \cos(\lambda_{m1}(\rho)) \]

\[ s_{\mu} = \left[ \frac{1}{n^2} k_{\mu} + \frac{n^2 - 1}{n^2} k_{\mu} \left( 1 - \frac{1}{\varphi_1} \xi_{m1}^2 \right) \right] \xi_{m1} \cosh(\xi_{m1}(\rho)) \]

\[ s_{\mu} = k_{\nu} \lambda_{m2} \cos\left( \lambda_{m2}(1 - \rho) \right) \]

In order to get unequal zero solutions of \( X \), ordering \( \det S = 0 \), the value of \( \beta_m \) can be obtained. Substituting for every value of \( \beta_m \) in Eq. (38), and ordering, \( a_{m1} = 1 \) the corresponding values of \( c_{m1} \) and \( c_{m2} \) are obtained.

Similar to PTIB, we get:

\[
A_m = \frac{(n^2 - 1)m_1 \int_0^h u_i g_{m1}(z)dz + n^2 m_2 \int_h^\infty u_i g_{m2}(z)dz}{(n^2 - 1)m_1 \int_0^h g_{m1}(z)dz + n^2 m_2 \int_h^\infty g_{m2}(z)dz} = \frac{(n^2 - 1)m_1 u_c W_{m1} + n^2 m_2 u_c W_{m2}}{(n^2 - 1)m_1 W_{m1} + n^2 m_2 W_{m2}} \tag{41}
\]

where:

\[ W_{m1} = \int_0^h g_{m1}(z)dz \]

\[ = a_{m1} \left( 1 + \frac{1}{\varphi_1} \lambda_{m1}^2 \right) \frac{H}{\lambda_{m1}} \left[ 1 - \cos(\lambda_{m1}(\rho)) \right] + c_{m1} \left( 1 - \frac{1}{\varphi_1} \xi_{m1}^2 \right) \frac{H}{\xi_{m1}} \left[ \cosh(\xi_{m1}(\rho)) - 1 \right] \]

\[ W_{m2} = \int_h^\infty g_{m2}(z)dz = a_{m2} \frac{H}{\lambda_{m2}} \left[ 1 - \cos(\lambda_{m2}(1 - \rho)) \right] \]

\[ W_{m3} = \int_0^h g_{m1}(z)dz = \frac{1}{2} a_{m1} \left( 1 + \frac{1}{\varphi_1} \lambda_{m1}^2 \right) \left[ h_1 - \frac{H}{2\lambda_{m1}} \sin(2\lambda_{m1}(\rho)) \right] \]

\[ + \frac{1}{2} c_{m1} \left( 1 - \frac{1}{\varphi_1} \xi_{m1}^2 \right) \left[ \frac{H}{2\xi_{m1}} \sinh(2\xi_{m1}(\rho)) - h_1 \right] \]

\[ + 2a_{m1} c_{m1} \left( 1 + \frac{1}{\varphi_1} \lambda_{m1}^2 \right) \left( 1 - \frac{1}{\varphi_1} \xi_{m1}^2 \right) \frac{1}{\lambda_{m1}^2 + \xi_{m1}^2} \]

\[ \times \left[ \frac{\xi_{m1} H \sin(\lambda_{m1}(\rho)) \cosh(\xi_{m1}(\rho)) - \lambda_{m1} H \lambda_{m1}(\rho) \sin(\xi_{m1}(\rho))}{\lambda_{m1} H \cos(\lambda_{m1}(\rho)) \cosh(\xi_{m1}(\rho)) - \lambda_{m1} H \lambda_{m1}(\rho) \sin(\xi_{m1}(\rho))} \right] \]

\[ W_{m4} = \int_h^\infty g_{m2}(z)dz = \frac{1}{2} a_{m2} \left( (H - h_1) - \frac{H}{2\lambda_{m2}} \sin(2\lambda_{m2}(1 - \rho)) \right) \]

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The forms of the solutions for $U_1 (z)$, $U_2 (z)$, $\overline{U}_1$, $\overline{U}_2$, $\overline{U}_P$ and $\overline{U}_S$ for the pervious bottom (PTPB) are the same as those for the impervious bottom (PTIB). Here, only the overall average degree of consolidation for the whole thickness of soil defined by excess pore water pressure, is presented:

$$
\overline{U}_P = 1 - \frac{1}{u_0 H} \left( \int_0^H u_1 dz + \int_0^H u_2 dz \right) = 1 - \frac{1}{u_0 H} \sum_{m=0}^\infty A_m \left( W_{m1} + W_{m2} \right) e^{-\beta_m t} \quad (42)
$$

COMPUTATION PROCEDURE

A computer program has been developed for the consolidation of ground with partially penetrated vertical drains. The main steps for PTIB are listed as follows:

1. Input every parameter of the system.

2. Calculate $\beta_m$ from $\det S = 0$. Because of $\beta_m > 0$, $\beta_m$ can start from a small positive number (for example, 0.00001). Give a step length (for example, 0.001), $\beta_m$ will become larger, step by step. If the characteristic of positive and negative of $\det S$ is changed, there must be a root of $\beta$ in this step. Then, the precise $\beta_m$ can be obtained by progressively reducing the interval.

3. Calculate $c_{ml}$ and $b_{m2}$ from Eq. (22).

4. Calculate $A_m$ from Eq. (27).

5. Repeat 2 - 4, to obtain the next set of $\beta_m$, $c_{ml}$, $b_{m2}$ and $A_m$.

6. Control the precision of the series solution by Eq. (33). For example,

$$
\left\| \overline{U}_P(t = 0) \right\| = \left\| 1 - \frac{1}{u_0 H} \sum_{m=0}^\infty A_m \left( W_{m1} + W_{m2} \right) \right\| < 0.005
$$

7. Calculate the corresponding values by Eq. (29) ~ Eq. (34).

It is very easy to obtain the precise calculation. The convergence of the series is rapid and the calculation is very fast on a PC.
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DISCUSSION

An approximate solution for calculating the overall average degree of consolidation for the whole thickness of soil \( \bar{U} \) was proposed early by Hart et al. (1958) as follows:

\[
\bar{U} = \rho \bar{U}_r + (1 - \rho) \bar{U}_z, \tag{43}
\]

where: \( \bar{U}_r \) = average degree of consolidation for the section with vertical drains. The consolidation due to vertical flow and that due to radial flow can be obtained immediately from charts or calculated with relative ease from analytical formula. The consolidation due to combined radial and vertical flow can be obtained by Carrillo’s method (1942).

\( \bar{U}_z \) = average degree of consolidation of the section without vertical drains. For the impervious bottom (PTIB), the drainage distance is \( H \). For the pervious bottom (PTPB), the drainage distance should be changed to \( H/2 \).

\( \rho \) = penetration fraction.

Eq. (43) does not consider the well resistance or smear effect. Under the condition of not considering well resistance and smear effect, the comparison of overall average degree of consolidation obtained by the present solution to that by Hart, et al.’s approximate solutions is shown in Fig. 2. As the common measurements of ground with the sand drains with the impervious bottom (PTIB), \( n = 10 \) and \( H/d_a \) are selected in Fig. 2. For the curve calculated by Hart et al.’s solution, Barron’s solution (1948) and Terzaghi’s one dimensional consolidation solution are adopted to calculate the consolidation due to radial flow and that due to vertical flow, respectively. The condition of not considering well resistance, can be calculated by the computer program for the present solution. For the case in Fig. 2, the calculation result with, \( k_r = 10^7 k_h \), is the same as that with, \( k_r = 10^6 k_h \). Hence, for the case in Fig. 2, if the coefficient of permeability of the vertical drains is bigger than \( k_r = 10^6 k_h \), the vertical drains can be regarded as without well resistance. Eq. (43) does not consider the fact that the drainage condition of the section without vertical drains is improved. It is clearly shown in Fig. 2. In the late consolidation stage, the overall average degree of consolidation obtained by Hart, et al.’s approximate solution is smaller than that by the present solution.

The modifications for \( \bar{U}_r \) and \( \bar{U}_z \) of Eq. (43) were proposed by Zeng and Xie (1989) as:

\[
\bar{U} = \rho \bar{U}_r + (1 - \rho) \bar{U}_z, \tag{44}
\]

in which \( \bar{U}_r \) = average degree of consolidation for the section with vertical drains.
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\[
\overline{U}_{rs} = 1 - \frac{8}{\pi^2} e^{-\beta, t}
\]

(45)

\(\overline{U}_s\) = average degree for consolidation of the section without vertical drains.

\[
\overline{U}_t = 1 - \frac{8}{\pi^2} e^{-\beta, t}
\]

(46)

where:

\[
\beta_r = \frac{\beta_s}{(1 - \alpha \rho)^2}, \quad \beta_s = \frac{\pi^2}{4} \frac{1}{H^2} \frac{k_v}{m_w \gamma_w}, \quad \alpha = 1 - \sqrt{\frac{\beta_r}{\beta_{rs}}}
\]

\[
\beta_r = \frac{32}{r_s^2 (F + \pi G) m_w \gamma_w}, \quad \beta_{rs} = \beta_r + \beta_s, \quad G = \frac{k_h}{k_w} \left(\frac{H}{2r_w}\right)^2
\]

\(G\) is well resistance factor, \(\rho\) is penetration fraction.

The above expressions are for the impervious bottom (PTIB). If, for the pervious bottom (PTPB), the drainage distance of the above expressions \(H\), should be changed to \(H/2\).
The terms \( \alpha \) and \( \beta' \) are applied to reflect the fact that the drainage condition of the section without vertical drains is improved due to the installation of vertical drains. Eq. (44) can also be considered for the well resistance or smear effect.

However, the drainage distance \( (H) \) in the well resistance factor \( (G) \) equals to the whole thickness of soil layer. The influence of well resistance on consolidation, is bigger than the actual condition. Hence, the overall average degree of consolidation obtained by Zeng and Xie’s approximate solution, is smaller than that by the present solution. The above analysis is also clearly shown in Fig. 3. Under the condition of considering well resistance, the comparison of overall average degree of consolidation obtained by the present solution to that obtained by Zeng and Xie’s approximate solutions, is shown in Fig. 3 for \( n = 10 \) and \( H/d_w = 100 \).

Moreover, only when \( \bar{U}_n > 30\% \) and \( \bar{U} \gamma = 30\% \), Eq. (45) and Eq. (46) are found, respectively. Ordinarily, the average degree of consolidation of the section without vertical drains achieves 30\% whereas the average degree of consolidation of the section with vertical drains is near to 100\%.

The overall average degree of consolidation under different well resistance and under different penetration fraction are shown in Fig. 4 and Fig. 5, respectively. It can be shown that the well resistance and the penetration fraction have significant influence on the consolidation of the system.

The consistent definition of consolidation of ground with partially penetrated vertical drains is assumed in all the above figures. It is to be noted that \( c_h = k_h / m_w \gamma_w \) in all the figures.

Fig. 3 Comparison of Model Prediction of Overall Average Degree of Consolidation to Zeng and Xie’s Approximate Solution
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Fig. 4  Overall Average Degree of Consolidation Under Different Well Resistance

Fig. 5  Overall Average Degree of Consolidation Under Different Penetration Fraction
CONCLUSIONS

A semi-analytical solution for consolidation of ground with partially penetrated vertical drains is developed and presented. The solution considers two kinds of drainage conditions, one for the impervious bottom (PTIB) and the other for the pervious bottom (PTPB). The main steps of computation procedure are presented. Based on the analysis and discussion, the following conclusions can be drawn:

1. Hart, et al.'s approximate solution does not consider the fact that the drainage condition of the section without vertical drains is improved. In the late consolidation stage, the overall average degree of consolidation obtained by Hart, et al.'s approximate solution is smaller than that by the present solution.

2. In Zeng and Xie's approximate solution, the influence of well resistance on consolidation, is bigger than the actual condition. The overall average degree of consolidation obtained by Zeng and Xie's approximate solution is smaller than that by the present solution.

3. The well resistance and the penetration fraction of vertical drains have significant influence on the consolidation of the system.

REFERENCES


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APPENDIX

Solution for Section with Vertical Drains

After some mathematical processing, the partial differential equations only containing $u_w$ and $u_1$ and relation of $u_w$ and $u_1$ are obtained as follows (Tang and Onitsuka, 1997a):

\[
\frac{k_{vl}}{m_{vl} \gamma_w} \frac{\partial^4 u_w}{\partial z^4} - \frac{\partial^3 u_w}{\partial z^3 \partial t} - \frac{k_{hi}}{m_{hi} \gamma_w r_v^2 F} \left[ 1 + \frac{k_{hi}}{k_w} \left( n^2 - 1 \right) \right] \frac{\partial^2 u_w}{\partial z^2} + \left( n^2 - 1 \right) \frac{2}{r_v^2 F} \frac{k_{hi}}{k_w} \frac{\partial u_w}{\partial t} = 0 \quad (A1)
\]

\[
\frac{k_{vl}}{m_{vl} \gamma_w} \frac{\partial^4 u_1}{\partial z^4} - \frac{\partial^3 u_1}{\partial z^3 \partial t} - \frac{k_{hi}}{m_{hi} \gamma_w r_v^2 F} \left[ 1 + \frac{k_{vi}}{k_w} \left( n^2 - 1 \right) \right] \frac{\partial^2 u_1}{\partial z^2} + \left( n^2 - 1 \right) \frac{2}{r_v^2 F} \frac{k_{hi}}{k_w} \frac{\partial u_1}{\partial t} = 0 \quad (A2)
\]

\[
\frac{\partial^2 u_w}{\partial z^2} = -\left( n^2 - 1 \right) \frac{2}{r_v^2 F} \frac{k_{hi}}{k_w} \left( u_1 - u_w \right) \quad (A3)
\]

Assuming a solution by separation of variables for Eq. (A1) as:

\[ u_w(z,t) = Z_w(z) T_v(t) \quad (A4) \]

Substituting Eq. (A4) into Eq. (A1) yields:
\[
\frac{k_{v_1}}{m_{v_i} \gamma_w} Z^-_w - \frac{k_{h_1}}{m_{v_i} \gamma_w} \frac{2}{r_s^2 F k_w} \left[1 + \frac{k_{v_1}}{k_w} (n^2 - 1)\right] Z^-_w = -\frac{T_i^*}{T_i} = \beta_i \tag{A5}
\]

where \(\beta_i\) is a positive arbitrary constant. For this type of heat conduction partial differential equations, \(\beta_i\) must be positive.

Also, Eq. (A5) gives:

\[
\frac{k_{v_1}}{m_{v_i} \gamma_w} Z^-_w - \left\{\frac{k_{h_1}}{m_{v_i} \gamma_w} \frac{2}{r_s^2 F} \left[1 + \frac{k_{v_1}}{k_w} (n^2 - 1)\right] - \beta_i\right\} Z'_w - (n^2 - 1) \frac{2}{r_s^2 F k_w} k_{h_1} \beta_i Z_w = 0 \tag{A6}
\]

Eq. (A6) is a linear fourth-order ordinary differential equation with constant coefficients. The corresponding algebraic characteristic equation is:

\[
\Lambda \chi_i^4 + \Xi \chi_i^2 + \Theta = 0 \tag{A7}
\]

where:

\[
\Lambda = \frac{k_{v_1}}{m_{v_i} \gamma_w}, \quad \Xi = \left\{\frac{k_{h_1}}{m_{v_i} \gamma_w} \frac{2}{r_s^2 F} \left[1 + \frac{k_{v_1}}{k_w} (n^2 - 1)\right] - \beta_i\right\}, \quad \Theta = -(n^2 - 1) \frac{2}{r_s^2 F k_w} k_{h_1} \beta_i
\]

Eq. (A7) is quadratic in \(\chi_i^2\).

In the above equation, \(\Lambda > 0\) and, \(\Theta < 0\) then, \(\sqrt{\Xi^2 - 4 \Lambda \Theta} > \Xi\), hence, the following two equations are founded:

\[
\chi_i^2 = -\Xi + \sqrt{\Xi^2 - 4 \Lambda \Theta} > 0 \tag{A8}
\]

\[
\chi_i^2 = -\Xi - \sqrt{\Xi^2 - 4 \Lambda \Theta} < 0 \tag{A9}
\]

Based on the theory of higher-order linear ordinary differential equations, Eq. (A8) has two opposite real roots, denoting as \(\xi_{m_i}/H\) and \(-\xi_{m_i}/H\). Corresponding to these two real roots, the solution of Eq. (A6) has \(c_{m_i} \sinh \left[\xi_{m_i} (z/H)\right] + d_{m_i} \cosh \left[\xi_{m_i} (z/H)\right]\) as two terms. Eq. (A9) has two conjugate imaginary roots, denoting as \(\lambda_{m_i}/H\) and \(-\lambda_{m_i}/H\). Where \(\lambda_{m_i}\) is the real number. Corresponding to these two conjugate imaginary roots, the solution of Eq. (A6)
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has \( a_{m_1} \sin \left[ \lambda_{m_1} \left( z/H \right) \right] + b_{m_1} \cos \left[ \lambda_{m_1} \left( z/H \right) \right] \) as two terms. Hence, the general solution for \( u_w \) can be written as:

\[
u_w = \sum_{m=0}^{\infty} Z_{wm}(z) T_m(t) = \sum_{m=0}^{\infty} A_m g_{wm}(z) T_m(t) \tag{A10}\]

in which:

\[
g_{wm}(z) = a_{m_1} \sin \left( \lambda_{m_1} \frac{z}{H} \right) + b_{m_1} \cos \left( \lambda_{m_1} \frac{z}{H} \right) + c_{m_1} \sinh \left( \xi_{m_1} \frac{z}{H} \right) + d_{m_1} \cosh \left( \xi_{m_1} \frac{z}{H} \right) \tag{A11}\]

\[
T_m(t) = e^{-\beta_{m_1} t} \tag{A12}\]

where: \( a_{m_1}, b_{m_1}, c_{m_1}, d_{m_1} \) and \( A_m \) are arbitrary constants. \( a_{m_1} \) is included in \( A_m \).

Substituting Eq. (A10) into Eq. (A3), the general solution for \( \bar{u}_r \) is obtained,

\[
\bar{u}_r = \sum_{m=0}^{\infty} Z_{rm}(z) T_m(t) = \sum_{m=0}^{\infty} A_m g_{rm}(z) T_m(t) \tag{A13}\]

where:

\[
g_{rm}(z) = \left[ 1 + \frac{\lambda_{m_1}^2}{\varphi_1} \right] \left[ a_{m_1} \sin \left( \lambda_{m_1} \frac{z}{H} \right) + b_{m_1} \cos \left( \lambda_{m_1} \frac{z}{H} \right) \right] + \left[ 1 - \frac{\xi_{m_1}^2}{\varphi_1} \right] \left[ c_{m_1} \sinh \left( \xi_{m_1} \frac{z}{H} \right) + d_{m_1} \cosh \left( \xi_{m_1} \frac{z}{H} \right) \right] \tag{A14}\]

Substituting Eq. (A10) and Eq. (A13) into Eq. (9) and Eq. (10), respectively, \( b_{m_1} = d_{m_1} = 0 \) is obtained.

Hence, the solutions for \( u_w \) and \( \bar{u}_r \) can be expressed as:

\[
u_w = \sum_{m=0}^{\infty} A_m \left[ a_{m_1} \sin \left( \lambda_{m_1} \frac{z}{H} \right) + c_{m_1} \sinh \left( \xi_{m_1} \frac{z}{H} \right) \right] e^{-\beta_{m_1} t} \tag{A15}\]

\[
\bar{u}_r = \sum_{m=0}^{\infty} A_m \left[ a_{m_1} \left( 1 + \frac{1}{\varphi_1} \lambda_{m_1}^2 \right) \sin \left( \lambda_{m_1} \frac{z}{H} \right) + c_{m_1} \left( 1 - \frac{1}{\varphi_1} \xi_{m_1}^2 \right) \sinh \left( \xi_{m_1} \frac{z}{H} \right) \right] e^{-\beta_{m_1} t} \tag{A16}\]
Solution for Section without Vertical Drains

Assuming a solution by separation of variables for Eq. (11) as:

$$u_2(z,t) = Z_2(z)T_2(t)$$  \hfill (A17)

Substituting above equation into Eq. (11) yields:

$$\frac{k_{v2}}{m_{v2}r_w - Z_2} = \frac{T_2}{Z_2} = \beta_2$$  \hfill (A18)

where $\beta_2$ is a positive arbitrary constant.

The equation for $Z_2$ of Eq. (A18) is a linear second-order ordinary differential equation with constant coefficients. The corresponding algebraic characteristic equation is:

$$\frac{k_{v2}}{m_{v2}r_w} \chi_2^2 + \beta_2 = 0$$  \hfill (A19)

hence,

$$\chi_2^2 = -\frac{m_{v2}r_w}{k_{v2}} \beta_2 < 0$$  \hfill (A20)

The equation has two conjugate imaginary roots, denoting as $\lambda_{m2}/H i$ and $-\lambda_{m2}/H i$.

The general solution for $u_2$ can be written as:

$$u_2 = \sum_{m=0}^{\infty} Z_{m2}(z) T_{m2}(t) = \sum_{m=0}^{\infty} A_m g_{m2}(z) T_{m2}(t)$$  \hfill (A21)

where:

$$g_{m2}(z) = a_{m2} \sin \left[ \lambda_{m2} \left(1 - \frac{z}{H}\right) \right] + b_{m2} \cos \left[ \lambda_{m2} \left(1 - \frac{z}{H}\right) \right]$$  \hfill (A22)

$$T_{m2}(t) = e^{-\lambda_{m2}t}$$  \hfill (A23)

For the impervious bottom condition (PTIB), $z = H$, $(\partial u_2/\partial z) = 0$, hence, $a_{m2} = 0$. The solution can be expressed as:
CONSOLIDATION OF GROUND

\[ u_2 = \sum_{m=0}^{\infty} A_m b_{m2} \cos \left[ \lambda_{m2} \left( 1 - \frac{z}{H} \right) \right] e^{-\beta_{m2} t} \]  \hspace{1cm} (A24)

For the pervious bottom condition (PTPB), \( z = H \), \( u_2 = 0 \), hence, \( b_{m2} = 0 \). The solution of \( u_2 \) can be expressed as:

\[ u_2 = \sum_{m=0}^{\infty} A_m a_{m2} \sin \left[ \lambda_{m2} \left( 1 - \frac{z}{H} \right) \right] e^{-\beta_{m2} t} \]  \hspace{1cm} (A25)
LABORATORY MEASUREMENTS OF SMALL STRAIN STIFFNESS OF GRANITIC SAPROLITES

C.W.W. Ng\textsuperscript{1}, Y.F. Sun\textsuperscript{2} and K.M. Lee\textsuperscript{3}

ABSTRACT

Non-linear stress-strain characteristics and stiffness-strain relationships of sedimentary soils and sands at small strains have been reported by many researchers. However, research work on the behavior of residual soils at small strains has rarely been reported. A key aspect of small strain soil behavior is its influence on ground response around many underground structures attributed to the variation of its shear stiffness with shear strain level. In this paper, the use of three internal semiconductors to measure stress-strain response of completely decomposed granite (saprolites) at small strains is reported. It is found that the stress-strain characteristics and the stiffness-strain relationships are highly non-linear. Shear stiffness decreases significantly as shear strain increases. The behavior of the granitic saprolites at small strain does not seem to behave significantly different as compared with other sedimentary soils.

INTRODUCTION

Weathering and surficial geologic processes produce weathered and surficial deposits that are commonly referred to as “soil”. In tropical regions, weathering of primary minerals is more intense and occurs in greater depth than elsewhere. An important aspect of geotechnical engineering is the understanding of mechanical behavior of soils, for which engineering calculations and mathematical models for soil behavior are formulated.

The assumption of a linear stress-strain relationship of soil inside the state boundary surface has been applied to almost all geotechnical engineering practice for many years. Recently it has been widely accepted that the stress-strain relationship is highly non-linear even at small strains (Fig. 1), a conclusion emerged from recent advances in both laboratory (Burland and Symes, 1982; Jardine, et al., 1984; Clayton

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Note: Discussion is open until 1 March 1999. This paper is part of the Geotechnical Engineering Journal, Vol. 29, No. 2, December 1998. Published by the Southeast Asian Geotechnical Society, ISSN 0046-5828.
and Khatrushi, 1986; Atkinson and Sällfors, 1991; Goto, et al., 1991; and Ng, et al., 1995) and in situ measuring techniques (Abbiss, 1981; Tatsuoka and Kohata, 1994). Typical strain ranges for retaining walls, foundations and tunnels constructed in stiff soils such as stiff clays and dense sands are over the range of strains where there is the greatest variation of stiffness. For instance, shear strains developed around a 10 m deep excavation in a stiff clay (Fig. 2) vary with distance away from the excavation but are generally less than 0.5%. This small strain range cannot be accurately determined in a triaxial test with conventional external displacement measuring devices. Burland (1989) and Mair (1993) have described the importance of small strain stiffness on various engineering problems.

Fig. 1 Approximate Strain Limits for Soil Structures (Mair, 1993)
LABORATORY MEASUREMENTS OF SMALL STRAIN STIFFNESS

Fig. 2 Contours of Shear Strain around a 10 m Deep Excavation in a Stiff Clay (Ng and Lings, 1995)

Much of the laboratory and field measurements of small strain stiffness were conducted in Europe and Japan on sedimentary soils and soft rocks. Research on the small-strain stiffness of residual soils has rarely been reported in the Far East. In this paper, the use of some internal local semiconductors to measure the small strain stiffness of granitic saprolites from Hong Kong is described and the measured highly non-linear stress-strain behavior is discussed.

ERRORS IN CONVENTIONAL MEASUREMENTS OF SOIL STIFFNESS

Conventionally, the determination of soil stiffness of a triaxial sample is based on external measurements of displacement which include a number of extraneous movements. The important sources of error are illustrated in Fig. 3. Some of the deflections shown in the figure may be quantified by careful calibration, but large unaccountable errors remain due to:

1. The difficulty of trimming a sample to ensure that the end faces are perpendicular to the vertical axis of symmetry;
Fig. 3 Potential Errors Using External Measuring Devices
LABORATORY MEASUREMENTS OF SMALL STRAIN STIFFNESS

2. Seating problem in the connection between the load cell and the sample top cap; and

3. The inevitable "bedding down" at the ends of the sample, due to local surface irregulars or voids.

The true axial strains developed in a soil specimen can be easily masked by these movements, leading to a poor definition of stress-strain behavior of the material under test, particularly over the small strain range. Conventionally, shear strains are calculated from external volumetric and axial strain measurements. Errors in volumetric measurements could also be significant due to the compliance of piping and possible volumetric change by air trapped at connections. Therefore, most conventional triaxial tests tend to give apparent soil stiffness far lower than those inferred from field observations. To overcome the measurement problems encountered in the laboratory, various local strain or displacement measuring devices have been developed and improved in recent years.

HALL EFFECT SEMICONDUCTORS

The Hall Effect semiconductors used were developed by Clayton and his coworkers (Clayton and Khattrush, 1986; and Clayton, et al., 1989) to measure local axial and radial strains of triaxial specimens (Fig. 4 and Fig. 5, respectively). The basic principle of the devices is to measure voltage change of an electrically energized semiconductor plate moving in a magnetic field. Figure 4 shows a Hall Effect sensor mounted onto a soil specimen inside a triaxial cell. As the soil specimen deforms, voltage change will be induced in the electrically powered Hall Effect sensors in the magnetic field, provided by a pair of bar magnets. Once the induced voltage is detected, the local axial deformation or strain of the soil specimen can be determined by using a calibration curve.

Hall Effect sensors can be used for both axial and radial measurements. They are typically energized by Direct Current (DC) and they deliver a DC output that varies linearly with magnetic flux density over a specified range. The resolution of these devices can reach up to ± 0.002%.

SAMPLE PREPARATION AND TEST PROCEDURE

Sample Preparation

Natural soil specimens of Completely Decomposed Granite (CDG) were taken from Kowloon Bay using the Mazier sampling technique (GCO, 1990), which is recognized as the best sampling method available in Hong Kong to obtain the so-called
"undisturbed" samples in CDG. The sampling depth was between 34 m to 38 m (m.P.D.), located at about 40 m below the existing ground surface. The samples were covered with wax immediately and then delivered to the laboratory. A summary of the test specimens are given in Table 1. All the specimens were tested in a computer controlled triaxial stress path apparatus equipped with internal (local) Hall Effect measuring devices.
LABORATORY MEASUREMENTS OF SMALL STRAIN STIFFNESS

Table 1 Details of Test Specimens

<table>
<thead>
<tr>
<th>Sample identity</th>
<th>Depth (m)</th>
<th>Initial M.C. (%)</th>
<th>Final M.C. (%)</th>
<th>B value</th>
</tr>
</thead>
<tbody>
<tr>
<td>KBBH54s</td>
<td>35-36.1</td>
<td>19.1</td>
<td>21.4</td>
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<td>KBBH55s</td>
<td>35-36.1</td>
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<td>21.5</td>
<td>0.91</td>
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<tr>
<td>KBBH56s</td>
<td>35-36.1</td>
<td>21.1</td>
<td>19.8</td>
<td>0.96</td>
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<tr>
<td>KBBH61s</td>
<td>37-38.1</td>
<td>31.2</td>
<td>27.6</td>
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<tr>
<td>KBBH62s</td>
<td>37-38.1</td>
<td>34.5</td>
<td>30.8</td>
<td>0.98</td>
</tr>
<tr>
<td>KBBH71s</td>
<td>35.7-36.8</td>
<td>20.8</td>
<td>20.9</td>
<td>0.98</td>
</tr>
<tr>
<td>KBBH72s</td>
<td>35.7-36.8</td>
<td>21.3</td>
<td>20.9</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Note: M.C = Moisture content

(a) Plan view of collar
Spring-loaded hinge
Pollished hinge-pin
Aluminum ring

(b) Hall effect sensor
Brass container
Hall effect sensor
Electrical cable
Sliding block
Bar magnets
Adjustment screw
PTFE spacer

Fig. 5 Hall Effect Gage for Radial Strain Measurement (Clayton, et al., 1989)
NG, SUN and LEE

To minimize sample disturbance during preparation, appropriate sample cutting and setting techniques are critical. A high quality cutting gives a reasonably "flat" surface on the sample to minimize any bedding error and non-uniform stress concentration in the specimen. Similarly, a smooth operation on the mounting of the membrane and the setting of the sample could minimize disturbance to the specimen. All these are very important for small strain measurements. During the sample preparation, care was taken to ensure a high quality specimen for the test.

To improve the efficiency of back pressure saturation, carbon dioxide was used to displace air (mainly nitrogen) in the voids of a specimen before saturation because carbon dioxide was more soluble than nitrogen. The whole carbon dioxide saturation process generally took at least 48 hours to complete.

After displacing air in the voids of the specimen with carbon dioxide, the specimen was saturated with de-aired water. Back pressure, up to 300 kPa, was applied to ensure a high degree of saturation. The $B$ values of the tests obtained were generally higher than 0.9.

Test Procedure

For all triaxial tests conducted at the University, the specimens were initially consolidated isotropically before shearing. No side drains were used. Consolidation was carried out against an elevated back pressure between 200 kPa and 300 kPa to ensure saturation of the specimen. The specimen was consolidated to have an effective all round pressure of 200 kPa, which was estimated to be the mean effective stress in the field.

In order to obtain shear modulus at small strains, constant $p'$ stress path was adopted (i.e., increasing $q$) with $p'$ and $q$ being the mean effective stress and shear stress, respectively. During the drained shearing stage, stress control was adopted in all drained tests. The loading rate was applied slowly at a rate of 5 kPa to 10 kPa per hour. Measurements of strains were made using both conventional and local axial and radial strain measurements. In addition to external load measurements, the axial force was recorded by an internal low-range (3 kN) load cell. The instrumentation used in the tests are summarized in Table 2.

<table>
<thead>
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<th>Table 2 Details of Instrumentation in the Triaxial Apparatus</th>
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<td><strong>Conventional measurements</strong></td>
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<td>Axial load</td>
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<tr>
<td>Axial displacement</td>
</tr>
<tr>
<td>Radial strain</td>
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<tr>
<td>Volumetric strain</td>
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<tr>
<td>Pore pressure</td>
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</table>
LABORATORY MEASUREMENTS OF SMALL STRAIN STIFFNESS

Laboratory Test Results

Figure 6 shows a typical constant $p'$ stress path for the tests. Internal and external $q$ values were determined from the internal and external load cells, respectively. Generally, the difference between the internal and external load cell measurements is about 5%. The values obtained by using local small strain measuring devices are illustrated in Fig. 7, which compares the results from internal and external measurements of axial strains. It can be seen that the resolution of the internal local measuring device (Hall Effect transducers) is about 10 times higher than the conventional external measuring device (LVDT). Figure 8 shows the stress-strain characteristics of CDG. Two important points can be noted from this figure. Firstly, the stress-strain response measured by the internal measuring devices is much stiffer than the one obtained from the external measuring devices. Secondly, the stress-strain relationship of CDG is highly non-linear, even at small strains.

Figure 9 shows a typical measured results of normalized shear stiffness ($G/p'$) versus shear strain on a logarithmic scale. The term $G$ is the secant shear stiffness of soil. Conventionally, shear strain is determined from external measurements of axial strain (LVDT) and volumetric strain using an external volume gage. In addition to the conventional approach, shear strain could be calculated from local axial and radial strain measurements. A significant difference between the two approaches can be clearly seen in the figure. From the local measurements, strong non-linear behavior of the soil is evident. According to local strain measurements, $G/p'$ decreases from about 500 at 0.003% to about 180 at 0.1%. On the other hand, the $G/p'$ values measured using the conventional approach are very scattered, especially for shear strain less than 0.5%, within which the soil stiffness is very important in the context of tunnels and deep excavations (Fig. 1 and Fig. 2). Figure 10 summarizes the normalized shear stiffness versus shear strain relationship for all seven tests of CDG using the internal measuring devices. Although the results are somewhat scattered at very small strain range, it is

![Fig. 6 Typical Constant $p'$ Stress Path](image-url)
NG, SUN and LEE

Fig. 7 Comparisons Between Internal and External Measurements

External and Internal (kPa)

Axial strain %
Fig. 8 Typical Internal and External Stress-strain Characteristics
NG, SUN and LEE

Fig. 9 Typical Internally and Externally Measured Stiffness-Strain Relationships
Fig. 10 Normalized Shear Stiffness Versus Shear Strain for Granitic Saprolites
evident that the shear stiffness of the soil is highly non-linear and its stiffness decreases significantly as shear strain increases. The measured scattered results at very small strains could be attributed to the initial variations of moisture contents (Table 1).

Malone, et al (1997) compared the laboratory measurements reported in this paper with in situ self-boring pressuremeter tests conducted at the same Kowloon Bay site by the Geotechnical Engineering Office (GEO) of the Hong Kong Special Administrative Region. Their comparisons show good consistency between the test results obtained from the laboratory and from the field (refer to Fig. 11 of their paper). This gives the authors some confidence in the measured values reported in this paper. The highly non-linear soil stiffness measured in the laboratory clearly indicates that it would not be appropriate (although still safe) to use a single soil stiffness obtained at large strains for predicting ground movements for geotechnical designs.

CONCLUSIONS

1. Both conventional and local strain measuring devices have been used to measure stress-strain characteristics and shear stiffness of natural granitic saprolites. The test results show that conventional external measurements of axial and radial displacements contain errors which mask the initial stress-strain characteristics of the soil and invalidate their use in the determination of soil stiffness. Both local axial and radial displacement measuring devices have been successfully applied to natural granitic saprolites, which are commonly found in the Far East.

2. The measured stress-strain characteristics of the natural granitic saprolites are highly non-linear even at small strains (less than 0.01%) and the measured shear stiffness decreases substantially as shear strain increases. The highly non-linear soil stiffness measured in the laboratory clearly indicates that it would not be appropriate to use a single soil stiffness obtained at large strains for predicting ground movements for geotechnical designs.

ACKNOWLEDGMENT

The authors would like to thank the Geotechnical Engineering Office of the Hong Kong Special Administrative Region for providing soil specimens reported in this paper.

REFERENCES

LABORATORY MEASUREMENTS OF SMALL STRAIN STIFFNESS


BOOK REVIEW

Book Title: Site preparation for the new Hong Kong International Airport; design, construction and performance of the airport platform.
Authors: G.W. Plant, C.S. Covil, and R.A. Hughes.
Publisher: Thomas Telford Publishing.
£ 85 (HK$ 850 in Hong Kong through the Airport Authority)

After more than a decade of investigation and design, on 30 November 1992, the Provincial Airport Authority awarded a $ 9.041 billion contract for the formation of a platform more than 5 km long in the sea off Lantau’s northern coast. Construction of Hong Kong’s airport was now beginning in earnest. The contract required the winning and precise manipulation of 360 million m$^3$ of earth and rock material, equivalent in volume to 130 Great Pyramids, yet the job had to be completed in just 41 months. By comparison, the Great Pyramid was built in an estimated 30 years. More than a minute degree of uneven settlement of the finished platform would not be tolerated. This was a tall order and the reclamation scheme designers had accordingly insisted that the deep layers of soft soils underlying the seabed at the site be stripped out. Unfortunately, this meant that a total of 109 million m$^3$ of mud, inclusive of overburden at the marine borrow areas, would have to be dredged and dumped. There was no practical alternative and capacity simply had to be found for dredged airport mud at the Government’s open sea disposal areas. High grade rock from Chek Lap Kok (CLK) and its neighboring islands would be needed for sea walls, to protect the edges of the platform from typhoon wave attack, and for earthquake-resistant foundations about 4 km long and 500 m wide under the two runways. The pick of Hong Kong’s newly discovered marine borrow areas was commandeered to supply the 76 million m$^3$ of clean sand needed to make up the balance of soft fill.

But would the land and marine borrow areas yield the huge quantities of the various engineering materials needed? Was the massive mud dumping operation environmentally viable? Could the geotechnical engineer’s predictions of platform settlement be trusted? Might the built reclamation even suffer the kind of large collapses that had occurred elsewhere in Hong Kong?

This would be no normal engineering enterprise. By virtue of the quantities of material to be moved in the short contract period, it would be necessary for the contractor – a multi-national joint venture of more than a dozen firms – to put together the biggest fleet of dredgers ever assembled. The contractor would need to work CLK island like a huge open-pit mine, with super large and fast drills, loaders and trucks. Would this unprecedented land and marine operation be achievable in such a very tight program?

Six years on and all key questions have been answered. This extraordinary contract was successfully completed four months early on 2 January 1996. The platform is working well, compressing and settling on its deformable geological foundation as intended. A fine achievement by its designer, contractor and the authority’s in-house supervisory team. One might say the CLK solution represents the ‘state-of-the-art’ and
the product of what the local civil engineering profession has learnt about the marine reclamation over forty years, starting from Kai Tak in the 1950s and going through the new town and container terminal reclamation.

It is therefore fitting and of great benefit to the profession that the airport platform case history should have become the subject of a new book, from Thomas Telford Publishing. The book covers design, construction and performance and comprises 19 chapters, a technical glossary, more than 350 references, and an index. It is well illustrated, with 193 line drawings, 39 black and white photographs, and 43 color illustrations. Written whilst the project was underway and published just in time for airport opening, the book is an extraordinary accomplishment by the authors, all busy people intimately involved in the job. And with its degree of fine detail on design and remarkable extent of early disclosure about performance, what a splendid example the Airport Authority has set for other client organizations.

The volume documents fully the design philosophy for the platform, (deciding the dredge levels, the distribution of the different types of engineered fill, predicting settlement) and gives the reasoning underlying the key decisions. Starting from 1982, the book charts the process of gradual scientific discovery about the natural ground at the site, recreating the environments of deposition of the soils over the last million years to give the vital geological models on which the designer relied. The authors set out in a simple way the engineering science on which the design is based (there are 15 pages on calculation of settlement) and describe the state-of-the-art instrumentation used to monitor performance. They compare the actual behavior of the platform against the predicted, i.e., in terms of the settlements and pore water pressure changes in the ground during construction. Primary consolidation settlements have generally taken place much faster than predicted but their magnitude is as expected. Future settlement to the year 2040 is estimated at 200 mm to 500 mm, of which half will be due to creep occurring in the generally 15 m to 20 m thick layers of rockfill and sand. There is much else to interest the maritime civil engineer (sea wall design, surveying, environmental impact mitigation, etc.) and the readers are given a full account of the methods employed by the contractor to realize the project (marine operations 37 pages, and operations 42 pages).

Future reclamation projects will be judged against the CLK benchmark. So if the reader is an engineer concerned with maritime reclamation he will need to have a copy of this book on his desk. The engineer is bound to ask himself, “How did they do it at CLK?” and “What worked at CLK?” For instance, “How was the settlement of the alluvial clay predicted?” “What allowance should be made for long-term creep settlement of sand and rockfill underwater?” “What bulking factors should be adopted for excavated rock?” “How well was the mud wave problem tackled?” “How effective was the surcharging and vibro-compaction?” or “What instrumentation worked successfully at CLK?” The reader will discover answers to all of these questions in the book. He will also find that Chapter Three gives the most comprehensive and authoritative treatment available of the geology of an engineering site in Hong Kong waters. A geotechnical engineer or engineering geologist shouldn’t attempt an interpretation of marine geology at an inshore site along the South China coast without first familiarizing himself with the CLK geological model.
At HK$ 850, this ‘state-of-the-art’ book with its mass of details is excellent value for money and probably less than the reader’s hourly charge-out rate. And, what’s more, all the royalties go to the Benevolent Fund of the Hong Kong Branch of the Lighthouse Club: a charity which donates money to the families of workers killed or injured in the local construction industry. So, my advise is, get yourself a copy of this useful book quickly, before it goes out of stock. It is bound to become a collector’s item.

Andrew Malone
Honorary Professor
Department of Earth Science
University of Hong Kong
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