ESTIMATING FUNDAMENTAL PERIOD OF SOIL PROFILES

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SYNOPSIS

A method for estimating the fundamental period of soil profiles is presented. The profile may have either stepped or continuous variation of soil properties along the depth. Vibration is assumed to be in shear mode. The profile is assumed to be divided into two segments. The stiffness matrix of a segment is obtained by recognising the soil layers in it to be in series and the mass matrix is obtained by assuming the mode shape to be linear within a segment. An eigenvalue problem of small size (2 × 2) results irrespective of the number of layers. A set of numerical examples are solved to demonstrate the accuracy of the method at minimal computation effort.

INTRODUCTION

Fundamental time period of soil profiles needs to be investigated in several situations. Vibration problems in geotechnical engineering involve energy loss due to material damping as well as radiation damping. It has been established that for a soil profile on rigid base rock the radiation damping at frequencies below the natural frequency of the soil profile is absent. This results in enhanced machine responce at operating frequencies below the natural frequency of the soil profile. Studies and observations have also indicated that excessive damage due to earthquakes occurs when the fundamental period, T, of the soil profile is closer to that of super structure. Simple procedures for estimating time periods are given (Dobry, et al 1976; Dobry & Gazetas 1985). Simple procedures though generally approximate, offer some advantages (Dobry & Gazetas 1985). These enable parametric studies to be carried out rapidly when soil properties cannot be evaluated accurately and a range of values of soil properties is to be assumed. These also serve as a check on the results of elaborate computer softwares. Further these help in developing engineering insight by recognising the essential features of a problem. Rigorous solutions (Dobry, et al 1976; Dobry & Gazetas 1985) are available for a soil layer having uniform properties as well as for a soil profile having specified continuous variation of soil stiffness with depth. For a soil profile having a number of soil layers, two commonly used methods

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are : (1) Rayleigh solution (Clough & Penzein 1975) and (2) two layer solution (Dobry, et al 1976). The former requires iterations and the later involves reading a design graph repeatedly.

Herein, a simple method for estimating T of a multilayer soil profile is presented. It can also be used for soil profiles having any continuous variation of soil properties by replacing the actual soil profile by a soil profile with stepped variations in properties. The stiffness matrix and the mass matrix based on an assumed displacement variation are obtained with respect to lateral deflection at two levels resulting in an eigenvalue problem of small size (2 × 2), the solution of which yields an accurate estimate of the time period. The method requires minimal computational effort and yields quite accurate estimate of T.

METHOD

Fig. 1 shows a multilayer soil profile. The profile depth is divided into two segments of about equal height. The top edge of segment 1 coincides with the top or bottom edge of a layer nearest the mid-height of the profile. Alternatively the profile may be divided into two equal halves. The top edge of segment 1 would not generally coincide with an edge of a layer nearest the mid-depth. In such a case this layer is divided into two portions with one portion of the layer in segment 1 and the another portion in segment 2. In either case it is then possible to determine the segment mass and stiffness properties in terms of the stiffness and mass properties of the layers contained in it. It is further assumed that vibration takes place in the shear mode. The stiffness and mass characteristics of the profiles are obtained w.r.t. lateral deflactions at level 1 and 2.

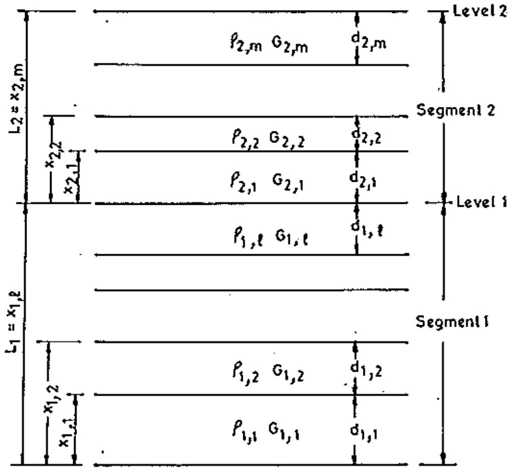

Fig. 1 A Multilayered Soil Profile.

The force per unit area k_i required to cause a relative displacement between the top level and bottom level of ith segment is obtained by recognising the layers in the segment to be in series and is given by

$$k_i = \frac{1}{\Sigma\ (d_{ij}/G_{ij})} \qquad (1)$$

in which G_{ij}, d_{ij} are shear modulus and depth of jth layer in the segment and summation extends over the number of layers in the segment.

The stiffness matrix [K] of the profile w.r.t. lateral deflections y(1) and y(2) at levels 1 and 2 is given by

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \qquad (2)$$

The mass matrix of a segment is obtained on the assumption that the mode shape is linear within the segment (Fig. 2). The matrix of shape functions $[N]_i$ for the ith segment is given by

$$[N]_i = [\ 1 - x/L_i \quad x/L_i\ ] \qquad (3)$$

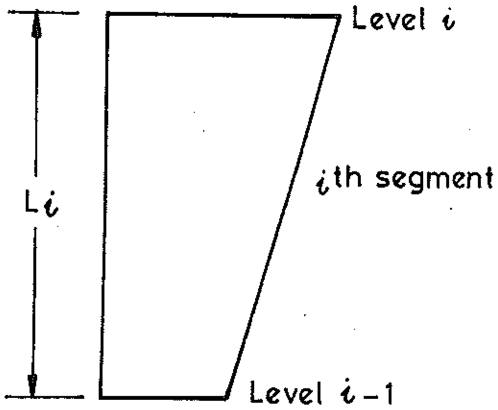

Fig. 2 Assumed Mode Shape in a Segment.

in which x is distance from bottom of a segment and L_i is length of ith segment.

The mass matrix $[M]_i$ of the segment is given by

in which ρ_i is varying mass density in the ith segment.

$$[M]_i = \int_0^{L_i} [N]_i^T \quad \rho_i \; [N]_i \quad dx \qquad (4)$$

Since the variation of ρ_i with x is in steps, the integration of Eq. 4 is carried out by discretising the limits of integration. The mass matrix $[M]_i$ can then be written as

$$[M]_i = \begin{bmatrix} M_i(1,1) & M_i(1,2) \\ M_i(2,1) & M_i(2,2) \end{bmatrix} \qquad (5)$$

where

$$M_i(1,1) = A_i - 2B_i + C_i \qquad (6a)$$

$$M_i(1,2) = M_i(2,1) = B_i - C_i \qquad (6b)$$

$$M_i(2,2) = C_i \qquad (6c)$$

$$A_i = \Sigma \rho_{i,j} \, a_{i,j}; \; B_i = \Sigma \rho_{i,j} \, a_{i,j} \, b_{i,j}/2 \, L_i$$

$$\text{and } C_i = \Sigma \rho_{i,j} \, a_{i,j} \, c_{i,j}/3 \, (L_i)^2 \qquad (7)$$

$$a_{i,j} = x_{i,j} - x_{i,j-1}; \; b_{i,j} = x_{i,j} + x_{i,j-1}$$

$$\text{and } c_{i,j} = (x_{i,j-1})^2 + (x_{i,j})^2 + x_{i,j-1} \, x_{i,j} \qquad (8)$$

In Egs. 7 and 8, summation extends over the number of layers in the seqment, $\rho_{i,j}$ = mass density of jth layer and $x_{i,j-1}$, $x_{i,j}$ = distance of bottom level and top level of jth layer respectively from the bottom of the segment. The terms $x_{i,j-1}$ in expressions for $a_{i,j}$, $b_{i,j}$ and $c_{i,j}$ are taken equal to zero for j = 1, i = 1,2.

The computational effort required in evaluation of the segment mass matrix by using Eqs. 6 - 8 depends on the number of layers in the segment. Noting that mass densities of layers do not differ much, considerable reduction in the effort results if an average mass density ρ_i is ascribed to all the layers in the segments and which is given by

$$\rho_i = \frac{\Sigma \rho_{i,j} \, d_{i,j}}{\Sigma d_{i,j}} \qquad (9)$$

in which the summation extends over the number of layers in the segment.

The terms of the segment mass matrix are now obtained simply as

$$M_i(1,1) = \rho_i L_i/3 \qquad (10a)$$

$$M_i(1,2) = M_i(2,1) = \rho_i L_i/6 \qquad (10b)$$

$$M_i(2,2) = \rho_i L_i/3 \qquad (10c)$$

The mass matrix of the soil profile [M] is given by

$$[M] = \begin{bmatrix} M_1(2,2) + M_2(1,1) & M_2(1,2) \\ M_2(2,1) & M_2(2,2) \end{bmatrix} \qquad (11)$$

The fundamental frequency ω is obtained by solving the standard free vibration problem [K] {y}, = ω^2 [M] {y}, in which [K] and [M] are given by Eqs. (2) and (11) respectively.

NUMERICAL EXAMPLE

To check the validity of the present method soil profiles with different distributions of soil properties along the depth are considered. In example 1 the soil profile has constant shear modulus G = 1.15×10^8 N/m², constant mass density = 1.15×10^3 kg/m³ and total depth, H = 20 m. In example 2, G varies linearly from 1×10^8 N/m² at the top to 2.8×10^8 N/m² at the bottom, = 1.15×10^3 kg/m³ throughout and H = 20 m. In example 3, G varies with depth as $G = G_0 (1 + az)^{1/2}$, ($G_0 = 1 \times 10^8$ N/m² is shearing rigidity at the top, a = 0.342, z = distance from the top), $\rho = 1.15 \times 10^3$ kg/m³ throughout and H = 20 m. Soil profile in example 4 and 5 are multilayered; the properties of layers are shown in tables 1 and 2.

Table 1 Properties of Soil Profile in Numerical Example 4.

Layer No.	Thickness (m)	Mass Density (kg/m³ × 10³)	Shear modulus (N/m² × 10⁷)
1 (bottom)	10.0	1.65	16.0
2	6.0	1.50	14.0
3 (top)	4.0	1.30	8.0

The values of periods, T for all the examples using Rayleigh method and the present method are shown in table 3. For types of variations of soil properties in example 1 - 3 explicit expressions or results for T in tabular/graphical form based on rigorous solutions are available. Values of T obtained from these are also included in Table 3. For evaluating integrals in Rayleigh method the depth of the soil profile is divided into a number of intervals (10, 20 and 40) and trapezoidal rule used. This method required 2 - 3 iterations for convergence to third significant figure. The results from the present method included in the tables are based on the use of simplified mass matrices (Eqs. 10).

Table 2 Properties of Soil Profile in Numerical Example 5.

Layer No.	Thickness (m)	Mass Density (kg/m^3 × 10^3)	Shear modulus (N/m^2 × 10^7)
1 (bottom)	5.0	1.75	60.0
2	5.0	1.65	40.0
3	4.0	1.66	30.0
4	3.0	1.68	20.0
5	2.0	1.50	18.0
6	1.5	1.56	16.0
7	2.5	1.50	14.0
8	3.0	1.72	9.5
9	2.0	1.66	8.0
10 (top)	2.0	1.31	8.3

Table 3 Time Period (in secs.) of Soil Profiles (Numerical Examples 1, 2, 3, 4 and 5).

Example No.	Method				
	Rigorous	Rayleigh		Present	
		No. of Intervals	Time Period	No. of Layers	Time Period
		10	0.247	2	0.247
1	0.253	20	0.249	-	-
		40	0.251	-	-
		10	0.181	2	0.180
2	0.183	20	0.182	4	0.180
		40	0.183	6	0.182
		10	0.174	2	0.174
3	0.177	20	0.176	4	0.176
		40	0.177	6	0.176
		10	0.240	3	0.244
4	-	20	0.242	-	-
		40	0.243	-	-
		10	0.271	10	0.280
5	-	20	0.271	-	-
		40	0.271	-	-

For example 1, in the present method the soil profile is divided into two layers of equal depth (one layer of 10 m in each segment). The resulting T = 0.247 secs. is about 2.37% in error. The periods using Rayleigh method are 0.247, 0.250 and 0.251 for 10, 20 and 40 intervals respectively.

For examples 2 and 3, using the present method, the soil profiles with continuous varying soil properties are divided into a number of layers of equal depth with the soil properties at the centre of each layer taken as constant across the depth of the layer.

For example 2, in the present method, the number of equal layers are 2, 4 and 6 (1, 2 and 3 layers in each segment) for the purpose of evaluation of segment stiffness matrices. The value of T obtained for 2, 4 and 6 layers are 0.180, 0.180 and 0.182 secs. respectively which are close to that (0.183 secs.) obtained using rigorous solution. Using Rayleigh method values of T obtained are 0.181, 0.182 and 0.183 secs. for 10, 20 and 40 intervals respectively.

For example 3 also, in the present method the number of equal layers chosen are 2, 4 and 6. The resulting values of T are 0.174, 0.176 and 0.176 secs., which are again close to that (0.177 secs.) obtained using the rigorous solution. Time periods obtained using Rayleigh method for 10, 20 and 40 intervals are 0.174, 0.176 and 0.177 secs. respectively.

In examples 4 and 5 layers have varying densities. In the present method weighted average density (Eq.9) is used to obtain segment mass matrices (Eq. 10). In example 4, layers 1 and 2 are assumed to be in segment 1 and layer 3 in segment 2. The value of T obtained is 0.244 secs. Which is close to that (0.243 secs.) obtained using Rayleigh method with the number of intervals equal to 40. Similarly in example 5, layers 1 - 3 are assumed to be in segment 1 and layers 4 - 10 in segment 2. Using the present method the period obtained is 0.280 secs. Which is about 3.3% higer than that 0.271 secs.) obtained using Rayleigh method with 40 intervals.

For example 4 and 5, T was also obtained using Eqs. 6 - 8 for determination of segment mass matrices instead of using simplified expressions (Eq. 10). Values of T obtained are 0.242 and 0.278 secs. respectively, which are close to 0.244 and 0.280 secs. obtained using simplified expressions for evaluation of segment mass matrices. Thus the use of simplified expressions for the evaluation of mass matrix suffices.

For practical problems the use of simplified expressions for the evaluation of segment mass matrices results in only small increase in computational work in evaluating T when the number of layers in the profile increases. The increase in computational effort is confined to evaluation of average mass densities and stiffness matrices of segments. The size of eigenvalue problem remains (2 × 2), irrespective of the number of layers in the profile.

For example 5, T is also obtained using 2 layer solution (Dobry, Oweis and Urza 1976; Dobry and Gazetas 1985). The value obtained is 0.300 secs. which is about 9.7% more than 0.271 secs. obtained using Rayleigh solution.

CONCLUSIONS

A simple and efficient method for estimating the fundamental time periods of soil profiles with either stepped variation or continuous variation of soil profiles has been presented. The method requires minimal computation work and yields quite accurate estimates of periods. Maximum error in the numerical examples solved is about 3.3%. The size of the eigenvalue problem to be solved is 2 × 2 irrespective of the number of layers in the segment. For practical problems the increase in computational work with increas in number of layers in the soil profiles is very small.

APPENDIX - CALCULATIONS FOR EXAMPLE 5

Stiffness and mass matrices are evaluated for a prism of cross - sectional area = 1 m^2.

Using Eq. (1) terms k_1 and k_2 (in N/m) are obtained as

$k_1 = 29.27 \times 10^6 \qquad k_2 = 7.461 \times 10^6$

Using Eqs. 10, terms (in kg) of segment mass matrices are obtained as

$M_1\,(2, 2) = 7882 \qquad M_2\,(1, 1) = 8411$
$M_2\,(1, 2) = M_2\,(2, 1) = 4205$
$M_2\,(2, 2) = 8411$

For the profile, [K] (in kN/m) and [M] (in kg) are

$$[K] = 10^6 \begin{bmatrix} 36.73 & -7.461 \\ -7.461 & 7.461 \end{bmatrix}$$

$$[M] = 10^3 \begin{bmatrix} 16.29 & 4.205 \\ 4.205 & 8.411 \end{bmatrix}$$

Solution of the eigenvalue problem $[k]\,\{y\} = \omega^2\,[M]\,\{y\}$ yields ω and hence T = 0.280 secs.

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