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ANALYSIS OF SOIL BEHAVIOR DURING EXCAVATION OF SHALLOW TUNNEL

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SYNOPSIS

When a shallow tunnel is excavated in soft ground, the ground above and on the both sides of the tunnel deforms towards the opening.

A simple analytical solution is proposed to predict :

1) The form of surface settlement distribution.

2) The form of settlement reduction from higher values at depth to lower values at surface.

3) The effect of the relative depth (H/D) of tunnel on the settlement ratio, S_{max}/S_c.

INTRODUCTION

During the last two decades a great deal of research has been performed mainly in Britain and the United States regarding the study of ground settlement due to tunnelling in soft ground. These can be classified in three categories :

a) Experimental works at the laboratory scale such as those performed by Atkinson et al (1975); Atkinson, Cairncross & James (1974); Atkinson & Orr (1976); Atkinson et al (1979).

b) Field observations and measurements such as those reported by Peck (1969); Attewell & Farmer (1974, 1975); Butler & Hampton (1975); Attewell (1977); Glossop & Farmer (1979); Lo Ng & Rowe (1984), Attewell et al by (1986).

c) Theoretical studies such as those by Atkinson et al (1975); Muir-Wood (1975); Attewell, Yeates & Selby (1986); and Sagaseta (1987).

Historically the problem of ground settlement due to underground excavation was fist assessed by Terzaghi (1942), but the first formulation for the surface settlement distribution was proposed by Peck (1969), and since then his formula, $S = S_{max} \cdot \exp\left(\frac{-X^2}{2i^2}\right)$, has been accepted as the basis for most of the further investigations.

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The purpose of this paper is to present a simple analysis of the behaviour of non-dilating ground through which a circular shallow tunnel is excavated. This analysis is based on the mathematical relationship that accounts for soil behaviour. The applicability of this analysis will be discussed in comparing the results with published data from experimental observations.

MECHANISM OF GROUND DEFORMATION

The dominant part of ground settlement above a shallow underground opening within a soil medium results from the subsidence of the opening roof. A relation between surface settelment and opening roof subsidence can be obtained if the change of ground volume is assumed to be negligible. This type of analysis cannot therefore be applied to cohesionless sandy - gravely soils or to ravelling ground.

When a circular opening with diameter D = 2a is excavated in a homogenous isotropic medium, if its radius is reduced by an amount Δa, then the radial movement ΔR of a point M at a distance R from the opening centre (Fig. 1a) is given by :

$$\pi a^2 - \pi (a - \Delta a)^2 = \pi R^2 - \pi (R - \Delta R)^2$$

then ignoring the second order terms (an error of less than 5%) gives :

$$\Delta R = a \frac{\Delta a}{R} \qquad (1\text{--}a)$$

If the depth of tunnel centre is H, the radial movement of a point N (Fig. 1b) on a line at an angle β to the vertical will be

$$\Delta R' = \frac{a}{H} \Delta a \,.\, \cos\beta \qquad (1\text{--}b)$$

and the vertical component of the movement is

$$S = \Delta R' \,.\, \cos\beta = \frac{a}{H} \Delta a \,.\, \cos^2\beta \qquad (2)$$

The vertical subsidence of the opening roof varies from a maximum value (S_c) at the crown to very small or nearly zero at the spring line (Fig. 1c). Furthermore, the downward movement of the soil, due to gravity, is evaluated as $W' = W.\cos\beta$ for any hypothetical slices containing the weight W, and inclined at an angle β (see Fig. 2a), thus the vertical subsidence of each point of the opening roof is estimated as:

$$\delta_c = S_c \,.\, \cos\beta \qquad (3)$$

where δ_c is the vertical downward movement of a point lying on the radius with angle β, and S_c is the maximum value which occurs at the crown of opening (Fig. 1c).

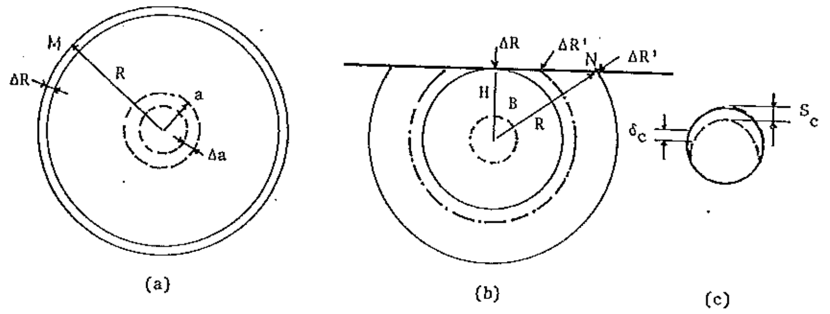


Fig. 1 (a) and (b) : Simple Assumptions Showing Deformation around an Opening; c) : Distribution of Subsidence inside an Opening.

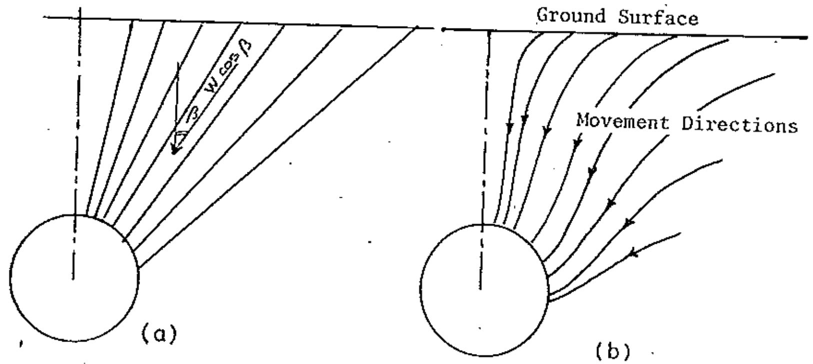


Fig. 2 Pattern of Soil Movements towards Opening; (a) : Simple Assumption (Hypothetical Slices), (b) : Experiment Based Results (after Hansmire & Cording, 1985).

The pattern of straight slices applied in Fig. 2a for simulating the soil movement is a simplified expression of the real movement of the medium (which can be observed either in the field or from laboratory tests), such as that illustrated in Fig. 2 – b.

Substitution of equation 3 into 2, results in

$$\left.\begin{aligned} s &= \frac{a}{H}\,\Delta a \cdot \cos^2\beta = \frac{a}{H}\,S_c \cdot \cos^3\beta, \text{ or} \\ S &= S_{max}\,\cos^3\beta = S_{max}\ /\ (1+X^2/H^2)^{3/2} \\ S_{max} &= S_c \cdot a/H \end{aligned}\right\} \quad (4)$$

The results of observations from either the field or laboratory model tests show that the surface settlement reduces to zero at a distance of about H (or $\beta = 45°$) from the centre line, and generally this distance varies for different conditions in the range of $30° < \beta < 75°$.

In order to bring the sense of Eq. (4) close to actual behaviour, this equation may be rearranged into a general form such as :

$$S/S_{max} = \cos^2\beta \cdot \cos(90\,\beta/\eta) \quad (5)$$

where the angle η is chosen from appropriate experiments.

For the usual cases in which the approximate extension of surface settlement is limited to $\pm$ H, then $\eta = 45°$, and equation (5) becomes

$$S = S_{max}\cos^2\beta \cdot \cos(2\beta) \quad (6)$$

The inflexion point of this equation is at $\beta = 26.8°$ or $x = 0.505$ H.

The graphical form of equation (5) with three values of η i.e. 45°, 57.5°, and 90° is shown in Fig. 3, where these curves are compared with the curve of a normal statistical function, $e^{-0.5(\frac{X}{i})^2}$ proposed by peck (1969) with different values of i.

This comparison indicates that each one of the curves of formula (5) can in general coincide with a curve of Peck's formula; for example the curve of formula 5 with $\eta = 45°$ is coincident with Peck's formula with i = 0.41 H.

Based on experimental data, the boundary of deformation around the opening may be idealised as a curve close to a parabola as shown in Fig. 4, the equation of this parabola is:

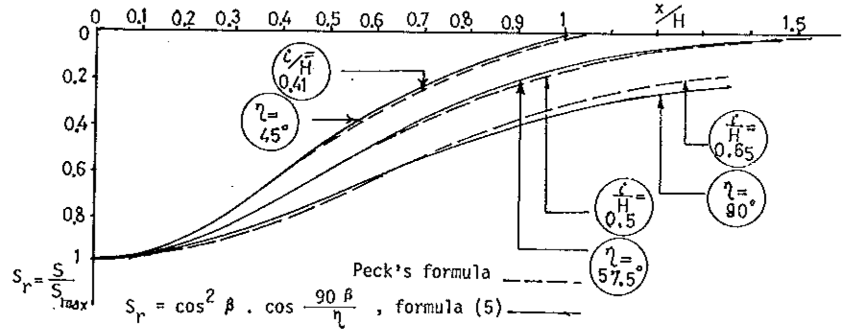


Fig. 3 Comparison of Statistical Normal Curve (Peck, 1969) with the Curves in Present Analysis.

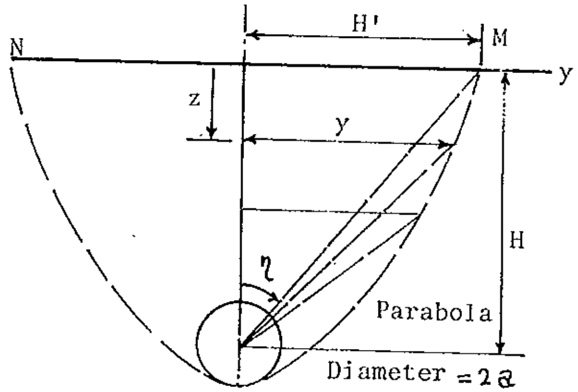


Fig. 4 Limitation of Settlement with the Region of Parabolic Boundary.

$$\lambda = H'\sqrt{1-Z/(H+a)} \quad (7)$$

and for $\beta = 45°$, H' = H.

From this equation the angle η for each level of Z (from the surface) is determined by

$$\tan\eta = y/(H-Z) \quad (8)$$

where for less cohesive soils the angle η would be normally less than 45°, and for cohesive Soils, more than or equal to 45°

RATIO OF MAXIMUM SURFACE SETTLEMENT TO THE MAXIMUM OF CROWN SETTLEMENT

The ratio of S_{max} (on the ground surface) to the maximum value of crown subsidence, S_c, as indicated by Egn. 4 contains some approximations, and to achieve a suitable relation it should be again referred to the experimental observations.

The pattern of deformation around real opening, and in model tunnel tests (see for example Atkinson and Potts, 1977) indicates that the type of inward deformation around the opening perimeter is similar to that shown in Fig. 5a. This is not obviously at variance with that shown in Fig. 1c, since that assumption was made for the vertical deformation only. Thus the corresponding soil deformation can be idealised in either of the forms in Fig. 5b, or Fig. 5c.

Equalising the whole volume of ground loss around the tunnel with the volume of soil displacement round the circle c (Fig. 5b), gives :

$$\pi a^2 - \pi \left(a - \frac{S_c}{2}\right)^2 = \pi \left(\frac{H+a}{2}\right)^2 - \pi \left(\frac{H+a-S_{max}}{2}\right)^2$$

from which, the following relation derives :

$$\lambda = \frac{S_{max}}{S_c} = \frac{2}{1+\frac{H}{a}} \qquad (9)$$

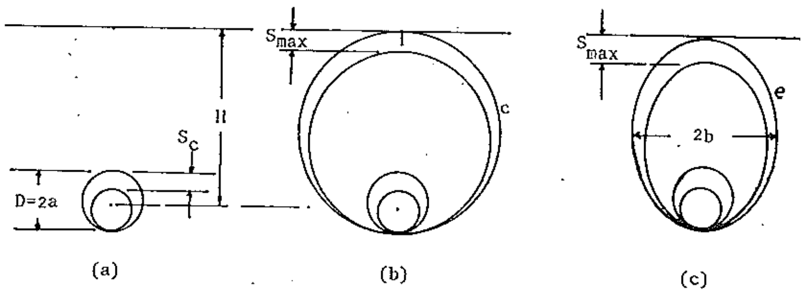


Fig. 5 (a) : Deformation of Opening Perimeter; (b) and (c) : Idealisation of Soil Displacement for the Points far of the Opening.

and for a depth of z below the surface :

$$\lambda = \frac{2}{1+(H-Z)/a} \quad , Z \le H-a \qquad (10)$$

On the other hand, assuming an elliptical pattern in Fig. 5c with diameters of H+a and $\sqrt{2a\,(H+a)}$ and equalising the volumetric displacements as for the circle gives :

$$\lambda = \frac{S_{max}}{S_c} = \frac{2}{\sqrt{2\,(1+H/a)}} \qquad (11)$$

The curves of Eqs. 9 and 11 are shown in Fig. 8 These two formulae can be accepted as the lower and upper bounds of λ as a function of relative depth (H/a).

COMPARISON OF RESULTS

In order to evaluate the reliability and suitability of this analysis, it should be compared with some appropriate results published by others. For this purpose the following type of data are discussed.

1) The form of surface settlement distribution.

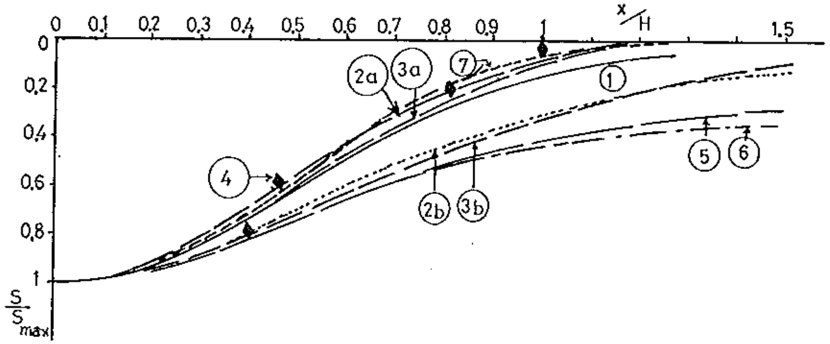
In Fig. 6 the results of some experimental observations are compared with the statistical function $\exp\left(\frac{-X^2}{2i^2}\right)$ proposed by Peck (1969), and the curves from the present analysis in the form of non-dimensional numbers.

This comparison indicates that the experimental data are in good agreement with the function $\cos^2\beta \cdot \cos\left(\frac{90\,\beta}{\eta}\right)$ so the general form of Eq.5 is an acceptable approximation of the surface settlement distribution.

2) The type of settlement variation along the vertical section.

Comparison of graphs indicating the variation of settlements in terms of the non-dimensional quantity $\lambda = S_{max}/S_c$ on the vertical axis of the tunnel are shown in Figs. 7a and 7b. For each case the actual dimensions of the tunnel have been used in calculating the settlement of some points, and the results adjusted to $S_c = 1$.

These figures show that the results of the present analysis are in good agreement with the measured quantities.



① Curve $S = S_{max} \cdot \exp\left(\frac{-X^2}{2i^2}\right)$, Peck (1969) for $i = H/2$

② Present Analysis :

$$S = S_{max} \cdot \cos^2\beta \cdot \cos\left(\frac{90\ \beta}{\eta}\right) \begin{cases} 2a, \eta = 50° \\ 2b, \eta = 75° \end{cases}$$

③ Based on Field Observations, H = 13.37m, D = 4.25m; Data form Glossop, 1978 (Ref. No. 6 and 8) — 3a after 23 days; 3b after 504 days

④ Based on the Measurement for Caracas Metro

⑤ Analytical Formula Proposed by Sagaseta

⑥ Finite Element Calculation (υ = 0.3)

(④, ⑤, ⑥: Data from Ref. No. 15)

⑦ Based on the Measured Values for a Tunnel in London clay, H = 29.3m, $H/_D \approx 7$, Ref. No. 7

Fig. 6 Comparison between the Proposed Curve (Equation 5) and Experimentally and Analytically desived Curves Showing the Extension of Settlement Trough.

3) The settlement ratio $\lambda = \frac{S_{max}}{S_c}$ as a function of H/a is one of the important subjects which has not yet been studied mathematically (see for example Lo et al, 1984).

Fig. 8 shows curves of the lower and upper bounds of the non-dimendional quantity λ against the ratio H/a according to the present analysis (curves 1 and 2), the curve proposed by Lo et al (Curve No. 3) and some experimental data from different authors collected by Lo, et al (1984).

This comparison indicates that the lower and upper bound equations from the present analysis are able to show good correlation with the relation between $\lambda = S_{max}/S_c$ and the relative depth H/a of the opening.

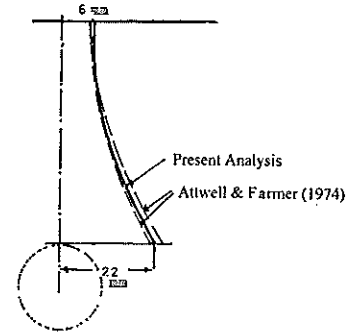


Fig. 7 (a) Comparison between the Results of the Present Calculation and Experimental Results from Attwell & Farmer (1974).

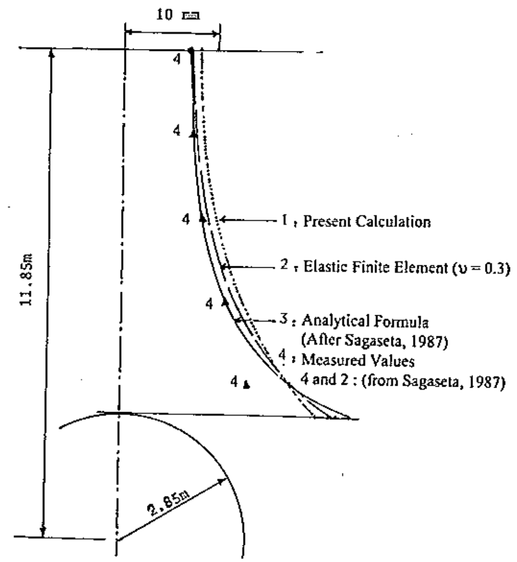
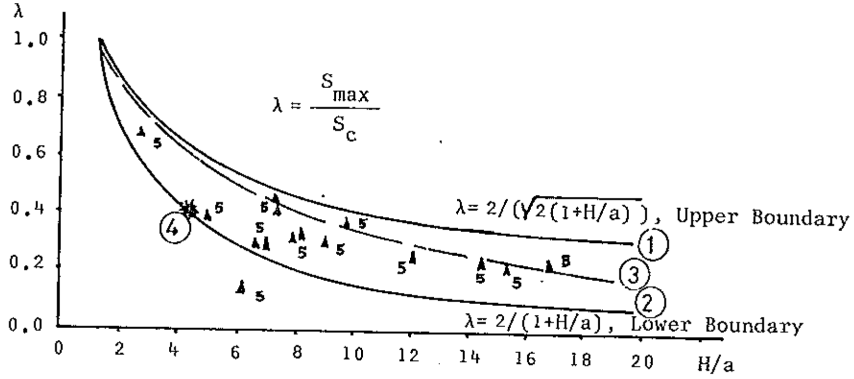


Fig. 7 (b) Variation of Settlement along the Vertical Asix for Caracas Metro.



① Upper Bound, Eq. 11 (present Analysis)

② Lower Bound, Eq. 9 (present Analysis)

③ Stiff Clay, proposed by Lo, et al (1984)

④ Finite Element Calculation for Caracas Metro (From Sagaseta, 1987)

⑤ Experiment Points Cited by Lo et al (1984)

Fig. 8 Comparison between the Results of Proposed Analysis and Experimental derived Results for λ against H/a.

CONCLUSION

A simple analysis has been proposed to simulate the deformation of soil ground through which a circular opening is excavated.

Comparison of the results from this calculation with the several experimental results showed that this analysis can serve as an acceptable solution for evaluating ground deformation characteristics such as settlement distribution, variation of settlement with depth, and reduction of settlement ratio with the relative depth.

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ERRATUM

Page 45, Volume 22 (June, 1991). The expression at (3) should read as follows:

$$\Sigma(E_L - E_R) - \Sigma N_p \sin \alpha - \Sigma S_m \cos \alpha = 0 \qquad \ldots (3)$$

with a horizontal passive thrust (P_P), $\Sigma(E_L - E_R) = P_P$