COMPUTATION OF PASSIVE EARTH PRESSURE BY A SIMPLIFIED METHOD OF SLICES

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SYNOPSIS

This paper describes the development of a method, based on a simplified method of slices, for evaluating passive earth pressure. Using the method, passive pressure coefficients for a vertical retaining wall with different slope geometries and mobilised wall friction angles have been calculated and compared with values obtained by other theoretical methods. Good agreement between the values calculated by the proposed method and those obtained by other theoretical methods was obtained.

INTRODUCTION

In retaining wall design, Coulomb's theory, the trial wedge method and the charts of Caquot & Kerisel (1948) are used for evaluating the passive earth pressure for a retaining wall (GCO, 1982). Both Coulomb's theory and the trial wedge method assume planar failure surfaces. This assumption can lead to substantial errors in the calculated values of the passive force because the failure surface for a passive failure is often curved, particularly when a large value of wall friction angle is mobilised (Graham, 1971; James & Bransby, 1970, 1971; Rowe & Peaker, 1965). The Caquot & Kerisel charts are however only applicable to granular soils and cases with simple geometries. For complex geometries, the passive force may be calculated using the circular arc method given by NAVFAC (1971). This method is very laborious for even simple ground conditions. It also appears that this approach has not been calibrated against other theoretical methods.

Janbu (1957) indicated that the generalised procedure of slices, which was developed for analysing slope stability, can also be applied to evaluate earth pressure and bearing capacity. However, this approach has not received much attention because the formulation given by Janbu is not sufficiently generalised: it is developed only for a vertical wall retaining horizontal ground. The limiting equilibrium formulation proposed by Shields & Tolunay (1973) is also limited to the same geometry.

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This paper describes the development of a method, based on a simplified method of slices, for evaluating passive earth pressure coefficients. In this method, the assumptions of zero interslice forces and overall force equilibrium, similar to those used in the Janbu Simplified Method (Janbu et al, 1956), are made. Using the method, passive pressure coefficients for a vertical retaining wall with different ground slopes and mobilised wall friction angles have been calculated and compared with values obtained by other theoretical methods.

SIMPLIFIED METHOD OF SLICES FOR EVALUATING PASSIVE EARTH PRESSURE COEFFICIENTS

The forces acting on a slice element in a passive earth pressure problem, in which an external thrust is applied to the soil, are shown in Fig. 1. In the figure,

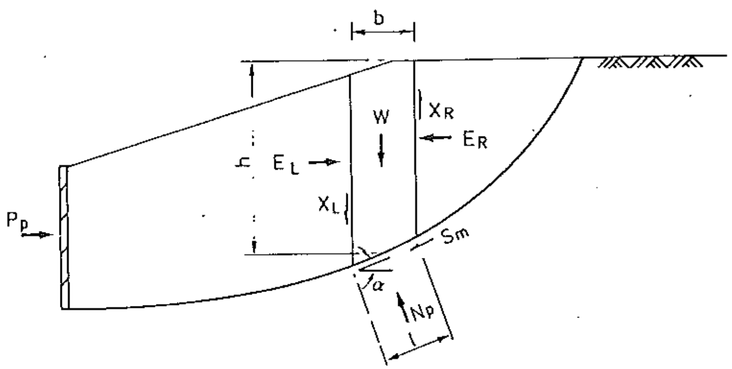

Fig. 1 Forces acting on a slice element in a retaining wall

Legend :

W	Total weight of the slice of width b and height h
N_p	Total normal force on the base of the slice over a length l
S_m	Shear force mobilised on the base of the slice
	$= c'\,l + (N_p - ul)\tan \varnothing'$
c′	Soil cohesion
Ø′	Angle of friction of soil
u	Pore water pressure
α	Angle between the tangent at the centre of the base of the slice and the horizontal
E_L, X_L	Interslice normal and shear forces on left and right sides of the slice
E_R, X_R	respectively
P_p	Passive thrust

E_L, X_L and E_R, X_R are the interslice normal and shear forces acting on the left and right sides of the slice respectively. A number of mathematical functions are available to describe the relationship between the interslice forces, notable of which are those given by Spencer (1967) and Morgenstern & Price (1965) (see also Fredlund & Krahn, 1977). However, for simplicity, zero interslice shear forces are assumed in the present formulation. It is acknowledged that not incorporating interslice shear forces in the formulation may over-simplify the actual situation. The results obtained could be less accurate, but they will err on the safe side: the calculated passive earth pressure coefficients will be lower than those derived assuming the presence of interslice shear forces, as there is no energy dissipation along the slice boundaries.

Consider the force equilibrium of a slice element in a retaining wall, as given in Fig. 1. By resolving forces vertically, the following equation is obtained,

$$W + X_L - X_R - N_p \, \mathrm{COS}\alpha + S_m \sin\alpha = 0 \qquad \ldots\ldots (1)$$

where

W	=	total weight of the slice of width b and height h
N_p	=	total normal force acting on the base of the slice over a length l
S_m	=	shear force mobilised on the base of the slice
	=	$c''\,l + (N_p\text{-ul}) \tan \varnothing'$
c′	=	soil cohesion
Ø′	=	angle of friction of soil
u	=	pore water pressure
X_L, X_R	=	interslice shear forces on left and right sides of the slice respectively
α	=	angle between the tangent at the centre of the base of the slice and the horizontal (Fig. 1)

Assuming zero interslice shear forces (i.e. $X_L = X_R = 0$) and substituting S_m into Equation (1), N_p becomes,

$$N_p = \frac{W + c'l \sin\alpha - ul \tan\varnothing' \sin\alpha}{\cos\alpha - \sin\alpha \tan\varnothing'} \qquad \ldots\ldots (2)$$

Forces acting on the slice element are then resolved horizontally for overall force equilibrium. The following expression is obtained,

$$\Sigma (E_L - E_R)\ \Sigma N_p \sin\alpha - \Sigma S_m \cos\alpha = 0 \qquad \ldots\ldots (3)$$

With a horizontal passive thrust (P_p), $\Sigma (E_L E_R) = P_p$. Hence,

$$P_p = \Sigma\{c'l\cos\alpha + (N_p\text{-}ul)\tan\varnothing'\cos\alpha\} + N_p\sin\alpha \qquad \ldots\ldots (4)$$

With wall friction, a passive thrust will have a vertical force component. Assuming that this vertical force component $P_p\tan\delta$, where δ is the wall friction angle, acts on the first slice, the expression for the total normal force acting on the base of the first slice becomes,

$$N_p = \frac{W + P_p\tan\delta + c'l\sin\alpha - ul\tan\varnothing'\sin\alpha}{\cos\alpha - \sin\alpha\tan\varnothing'} \qquad \ldots\ldots (5)$$

EVALUATION OF THE PROPOSED METHOD

Computer Program MJSM

Based on the formulation given above, a computer program (MJSM) has been developed to calculate the passive thrust for a retaining wall problem.

For the retaining wall problem, different failure surfaces have to be tried in order to obtain the minimum passive force. This approach is similar to a slope stability problem in which the critical failure surface giving a minimum factor of safety has to be determined. For the passive force calculations described in this paper, the suggestions of NAVFAC (1971) for the likely positions of the surfaces of sliding for the circular arc method have been used for the initial trial.

In the computer program, given the slope geometry, the material properties and the mobilised wall friction angle, a horizontal passive thrust (i.e. assuming no wall friction) is computed first. With the computed value, the total normal force acting on the base of the first slice is then revised to incorporate the vertical force component due to wall friction, which is calculated by multiplying the tangent of the mobilised wall friction angle and the computed horizontal thrust. The computation is repeated until the incremental value of the horizontal component of the passive thrust becomes less than 1% of the previously computed value.

Assessment of the MJSM Program for Passive Pressure Computation

The suitability of the MJSM program for passive pressure computation has been assessed by first calculating the passive pressure coefficients for a 10 m high vertical wall retaining sloping ground composed of a granular backfill and then comparing these with values obtained by other theoretical methods. Two materials, with angles of friction ($\varnothing'$) of 30° and 40° and a unit weight of 2 t/m^3 (mass density = 2 Mg/m^3) have been investigated. The analyses cover slope angles (β) of $0.4\varnothing'$, 0, $-0.4\varnothing'$ and $-0.8\varnothing'$ and wall friction angles (δ) of 0, $0.2\varnothing'$ and $0.5\varnothing'$. With

Table 1 Comparison of Passive Pressure Coefficients.

$\varnothing'$ (deg.)	$\beta/\varnothing'$	$\delta/\varnothing'$	δ (deg.)	Coefficients of Passive Pressure (K_p)		
				Simplified Method of Slices	Caquot & Kerisel	Coulomb
30	0.4	0	0	4.561	4.666	4.351
		0.2	6	5.571	5.740	5.687
		0.5	15	7.888	7.460	9.085
	0	0	0	2.998	3.000	3.000
		0.2	6	3.593	3.731	3.621
		0.5	15	4.545	4.849	4.977
	–0.4	0	0	2.146	1.821	2.066
		0.2	6	2.301	2.239	2.338
		0.5	15	2.696	2.909	2.882
	–0.8	0	0	1.285	0.864	1.296
		0.2	6	1.361	1.062	1.377
		0.5	16	1.512	1.380	1.538
40	0.4	0	0	9.192	9.956	8.994
		0.2	8	14.795	14.250	15.505
		0.5	20	28.800	22.496	53.082
	0	0	0	4.601	4.599	4.599
		0.2	8	6.221	6.750	6.351
		0.5	20	9.035	10.656	11.771
	–0.4	0	0	2.542	3.554	2.563
		0.2	8	3.062	4.368	3.110
		0.5	20	4.373	5.677	4.428
	–0.8	0	0	1.279	0.629	1.287
		0.2	8	1.389	0.900	1.407
		0.5	20	1.579	1.421	1.672

Legend : $\varnothing'$ = angle of friction of soil (degree)
β = ground slope angle (degree)
δ = wall friction angle (degree)

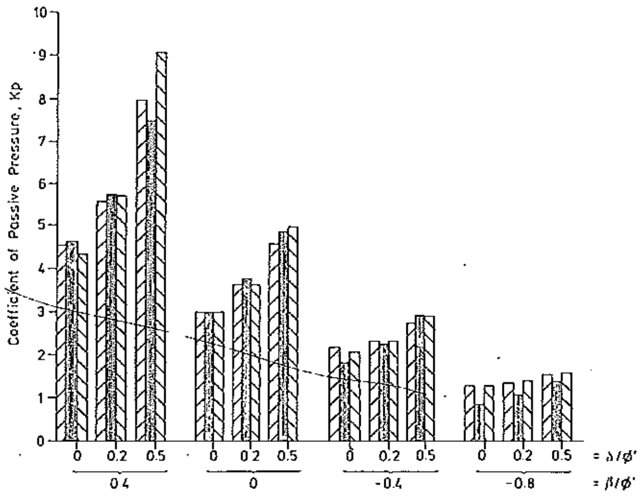

Fig. 2 Comparison of passive pressure coefficients for a cohesionless material with a friction angle of 30°.

Legend :

[hatched]	Simplified method of slices	Ø′	Angle of friction of soil
[hatched]	Coulomb	β	Ground slope angle
[hatched]	Caquot & Kerisel	δ	Wall friction angle

the exception of the case of horizontal ground and zero wall friction (which has a theoretically well-defined failure plane), trials of various failure surface geometries were necessary in order to obtain the minimum passive thrust, which was then converted to a passive pressure coefficient (K_p).

The results of the analyses, together with K_p values obtained using the Caquot & Kerisel charts and Coulomb's theory, are given in Table 1. A graphical comparison of these values is given in Figs. 2 and 3. The Caquot & Kerisel values are generally considered to be reasonably accurate as the combined logarithmic spiral and planar failure surfaces used are believed to model closely the actual situation. The calculated values by Coulomb's theory are included to illustrate the differences which may occur due to the assumption of planar failure surfaces.

As shown in Figs. 2 and 3, for a wall with sloping ground of 0.4Ø′, the passive pressure coefficients calculated by the proposed method agree closely with those by

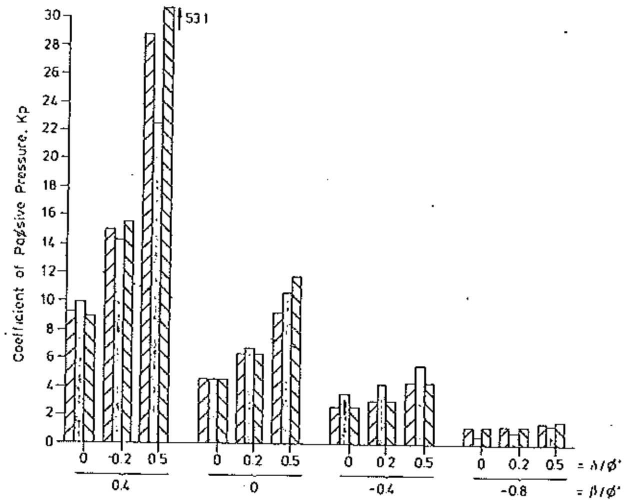

Fig. 3 Comparison of passive pressure coefficients for a cohesionless material with a friction angle of 40°.

Legend :

[hatched]	Simplified method of slices	Ø′	Angle of friction of soil
[hatched]	Coulomb	β	Ground slope angle
[hatched]	Caquot & Kerisel	δ	Wall friction angle

Caquot & Kerisel and Coulomb's theory when the mobilised wall friction angle is 0 or 0.2Ø′. For a higher wall friction angle of 0.5Ø′, the values given by the proposed method and Caquot & Kerisel are close, but the values calculated by Coulomb's theory are much higher. For the cases of a wall retaining horizontal ground and sloping ground of −0.4Ø′, the agreement among the values given by the proposed method, Caquot & Kerisel and Coulomb's theory is good. In some of these cases, values even lower than those by Caquot & Kerisel are obtained by the proposed method. When the ground slope angle behind the wall is −0.8Ø′, some discrepancies between the values given by the proposed method and Caquot & Kerisel are noted. The discrepancy becomes more marked when the mobilised wall friction angle is zero. The values obtained by the proposed method are, surprisingly, similar to those derived from Coulomb's theory for these cases.

Discussion

In order to locate the failure surface that gives the minimum passive force, different failure surfaces were tried. The geometries of failure surfaces found to give

$$\left.\begin{aligned}\theta_1 &= \frac{1}{2}(90+\varnothing') + \frac{1}{2}(\in + \beta)\\ \theta_2 &= \frac{1}{2}(90+\varnothing') - \frac{1}{2}(\in + \beta)\end{aligned}\right\} \quad \text{where } \in \; = \sin^{-1}\frac{\sin\beta}{\sin\varnothing'}$$

(after NAVFAC, 1971)

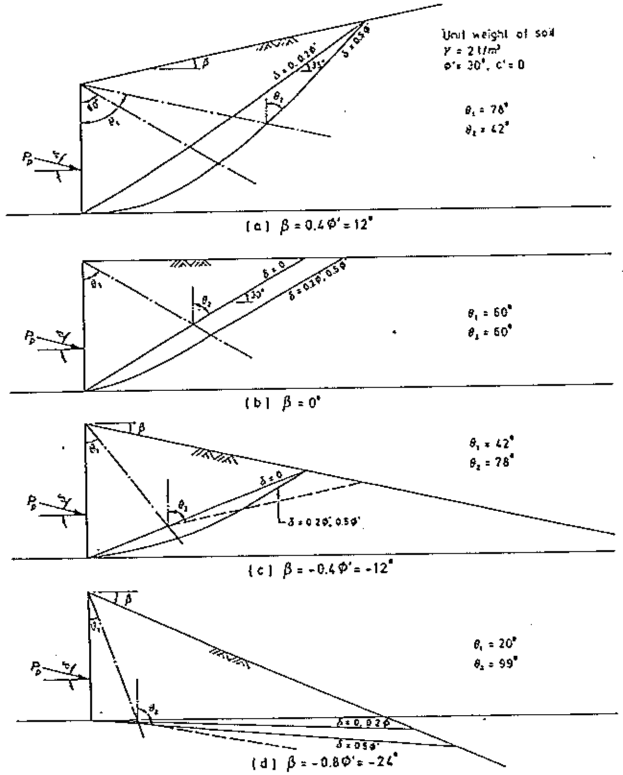

Fig. 4 **Geometries of critical failure surfaces which give minimum passive pressure coefficients for a cohesionless material with a friction angle of 30°.**

Legend :

β	Ground slope angle	P_p	Passive thrust
δ	Wall friction angle	$\varnothing'$	Angle of friction of soil

$$\left.\begin{aligned}\theta_1 &= \frac{1}{2}(90+\varnothing') + \frac{1}{2}(\in + \beta)\\ \theta_2 &= \frac{1}{2}(90+\varnothing') - \frac{1}{2}(\in + \beta)\end{aligned}\right\} \quad \text{where } \in \; = \sin^{-1}\frac{\sin\beta}{\sin\varnothing'}$$

(after NAVFAC, 1971)

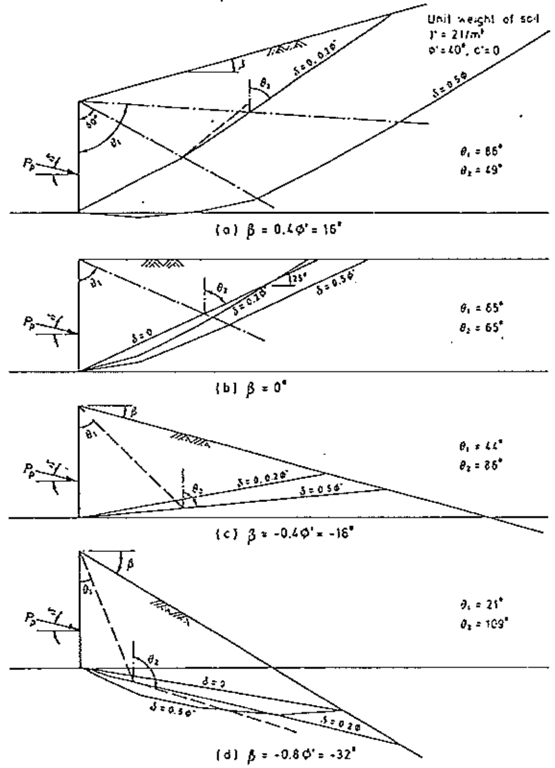

Fig. 5 **Geometries of critical failure surfaces which give minimum passive pressure coefficients for a cohesionless material with a friction angle of 40°.**

Legend :

β	Ground slope angle	P_p	Passive thrust
δ	Wall friction angle	$\varnothing'$	Angle of friction of soil

minimum passive forces for the cases considered are shown in Figs. 4 and 5. The angles Θ_1 and Θ_2 calculated using the equations given in NAVAC (1971) for the circular arc method are also included in the figures for comparison.

For a wall retaining horizontal ground, the failure surfaces are inclined at the theoretical angle of (45–Ø′/2) to the horizontal when the mobilised wall friction angle is zero. With wall friction, bi-planar failure surfaces were used. In the case of a wall with a ground slope of 0.4Ø′, bi-planar failure surfaces were used when the wall friction angle is 0 or 0.2Ø′, but curved failure surfaces were found to be necessary when the wall friction angle is 0.5Ø′. For a wall retaining a ground slope of –0.4Ø′ with a backfill friction angle of 30°, planar failure surfaces are satisfactory when there is no wall friction. Curved failure surfaces are necessary when wall friction exists. For the same wall retaining a backfill of friction angle of 40°, planar failure surfaces were adequate for all the angles of wall friction investigated. For a wall retaining a ground slope of –0.8Ø′, planar failure surfaces were used for Ø′ of 30°. For Ø′ of 40°, planar failure surfaces were used only for the two cases of wall friction angle of 0 or 0.2Ø′. A curved failure surface was necessary when the wall friction angle is 0.5Ø′.

It was found that the suggestions given by NAVFAC (1971) for the likely positions of the surfaces of sliding for the circular arc method are suitable for the initial trial in the search for the critical failure surface. As shown in Figs. 4 and 5, the failure geometries that give the minimum passive thrusts can be defined reasonably closely by the two angles Θ_1 and Θ_2 in most cases. In general, not more than ten trial surfaces were needed to obtain the minimum passive force. The time required for each trial including data preparation is approximately three minutes. If the calculations were to be done by hand using the circular arc method, it is estimated that each trial would take at least one hour.

For a wall retaining a ground slope of 0.4Ø′, curved failure surfaces were found to be necessary, particularly when the mobilised wall friction angle is large. This explains why the passive pressure coefficients calculated for such cases by Coulomb's theory, which assumes planar failure surfaces, are much higher than those computed using the program MJSM. In the case of a wall retaining a ground slope of –0.8Ø′, it appears that planar failure surfaces are adequate. For the se cases, the values calculated by Coulomb's theory are consistent with those given by theproposed method. It appears except for the case of a wall retaining a ground slope of 0.4Ø′ with a high wall friction angle of 0.5Ø′, Coulomb's theory gives results consistent with those obtained by the other two methods for those cases not involving curved failure surfaces.

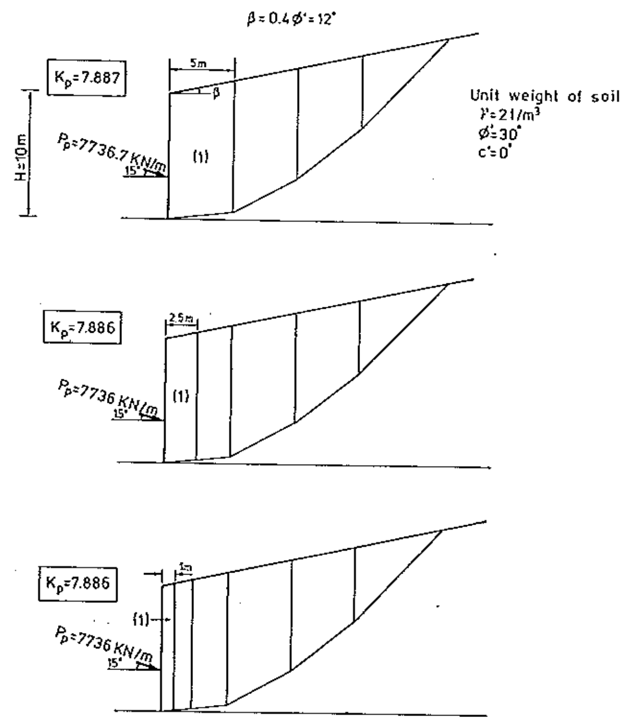

Fig. 6 – Effect of the width of first slice on the computed passive pressure coefficients for a wall retaining a ground slope of 0.4Ø′. (Ø′ = 30°)

Legend :

β	Ground slope angle	P_p	Passive thrust
δ	Wall friction angle	K_p	Coefficient of passive pressure
Ø′	Angle of friction of soil	c′	Soil cohesion
(1)	First slice	H	Height of wall

In the method proposed, the normal force acting on the base of the first slice is revised iteratively to incorporate the vertical force component due to wall friction. It is considered that the width of the first slice could have some influence on the computed results. To study this effect, two cases involving a wall retaining a ground slope of 0.4Ø′ and –0.8Ø′ were analysed. In the former case, an initial width of 5

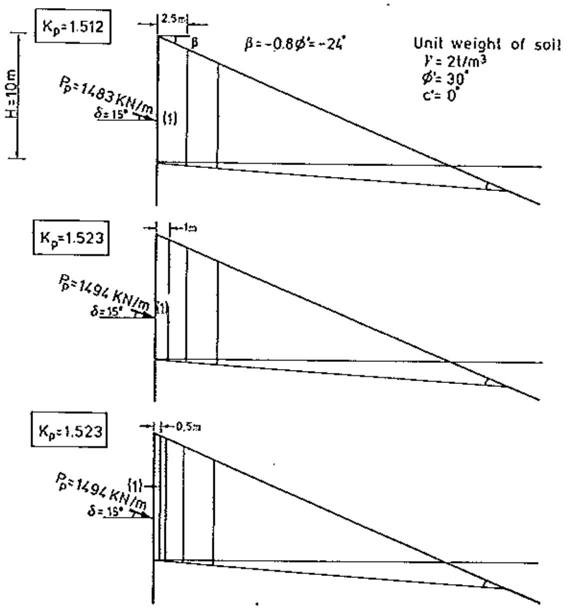

Fig. 7 · Effect of the width of first slice on the computed passice pressure coefficients for a wall retaining a ground slope of $-0.8\varnothing'$. ($\varnothing' = 30°$)

Legend :

β	Ground slope angle	P_p	Passive thrust
δ	Wall friction angle	$\varnothing'$	Angle of friction of soil
K_p	Passive pressure coefficient	H	Height of wall

m for the first slice was adopted in the analysis. This was subsequently reduced to 2.5 m, and then to 1 m. In the latter case, the width of the first slice was reduced from an initial value of 2.5 m to 1 m and then to 0.5 m. A backfill friction angle of 30° and a high wall friction angle of $0.5\varnothing'$ were used in both cases. The results, as summarised in Figs. 6 and 7, indicate that the computed passive pressure coefficients are insensitive to variations in the width of the first slice: almost identical results were obtained for the three different slice widths used.

CONCLUSIONS AND RECOMMENDATIONS

The conclusions and recommendations resulting from this study are:

(a) A simplified method of slices has been proposed for evaluating passive earth pressure. The method is based on overall force equilibrium assuming zero interslice shear forces. The method has been used to calculate passive pressure coefficients for a vertical wall backfilled with granular material and with a range of slope geometries and mobilised wall friction angles. The results have been found to compare well with the values obtained by Coulomb's theory and the Caquot & Kerisel charts. In theory, the proposed method, which is essentially a generalised method of slices, is equally applicable to cases where there are pore water pressures, surcharges, soil layering and complex slope geometries. However, it has only been checked for the simplest case of uniform granular material with zero water pressure.

(b) Curved failure surfaces were found to be necessary for a vertical wall retaining a positive ground slope of $0.4\varnothing'$ with a high mobilised wall friction angle. Coulomb's theory has limitation in this case as the theory assumes planar failure surfaces. However, it gives results consistent with those obtained by the proposed method and the Caquot & Kerisel charts for other cases not involving curved failure surfaces.

(c) In the case of a wall retaining a negative ground slope of $-0.8\varnothing'$, either planar or bi-planar failure surfaces would appear to be adequate. The passive pressure coefficients obtained from the Caquot & Kerisel charts are much lower than those calculated by the proposed method and Coulomb's theory. Therefore, the Caquot & Kerisel charts may have limitations for large negative ground slope values.

(d) For simple geometries and ground conditions, it is estimated that there is a saving in computation time of at least twenty times in using the computer program MJSM for evaluating passive pressure coefficients compared with the hand-calculated circular arc method given by NAVFAC (1971). For more complex problems, it is likely that there will be more substantial saving in computation time.

(e) The width of the first slice is found to have little effect on the computed passive pressure coefficients for the types of problems analysed in this study.

(f) For simplicity, the proposed method has been derived assuming zero interslice shear forces and overall force equilibrium. Although the results produced are in good agreement with those by other theoretical methods, further study is needed to incorporate interslice shear forces in the model to make it more realistic. Extension of the formulation to satisfy both the force and moment equilibrium is also recommended.

(g) This study focused primarily on the evaluation of passive earth pressure for a vertical retaining wall. Extension of the proposed method to the case of a sloping wall should also be considered.

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