VERTICAL VIBRATION OF PILES IN NON-HOMOGENEOUS SOIL

Fakhry Aboul-ella*

SYNOPSIS

A dynamic model for the vertical vibration of piles driven in soil, which has properties varying with distance from the pile, is presented. Soil layers around the pile are divided into rings of different properties. The effect of high strains around the piles and lack of bond between soil and piles can be studied using this model.

INTRODUCTION

In recent years, there has been a marked increase in interest in the dynamic behaviour of piles due to its effects on the behaviour of structures with soil-pile interactions (Aboul-ella 1984; Aboul-ella & Novak 1980). Novak and Aboul-ella (1978 a) have presented a model for the impedance functions (complex stiffnesses) of piles embedded in layered soil which assumes that the soil resistance (reactions) are those of the plane strain case and represents the pile by finite elements. The theory is based on the assumptions that the soil is bonded to the piles and is linearly viscoelastic. However, the region adjacent to the piles can have high strains which reduces the dynamic shear modulus and increases the damping of this region. This is a non-linear behaviour which can be studied, as well as the lack of bond between pile and soil, by assuming that each layer of soil surrounding the pile is composed of rings (Fig. 1) of different properties.

This paper presents the matrix formulation of the complex soil reactions of layers which can be modelled by soil rings. The soil properties, which can be adjusted according to the strain level, are assumed constant within each ring and different in individual rings. Each ring is homogeneous, isotropic and linearly viscoelastic with material damping of frequency independent hysteretic type. Hindy & Novak (1980) and Sheta & Novak (1980) have analyzed a case of a rigid cylinder surrounded by a massless thin ring followed by an infinite medium. The use of one massless ring around the pile is not sufficient to study soil non-

* Associate Professor, King Faisal University, College of Architecture and Planning, Dept. of Building Sciences and Tech. Dammam, P.O BOX 2397, Saudi Arabia.

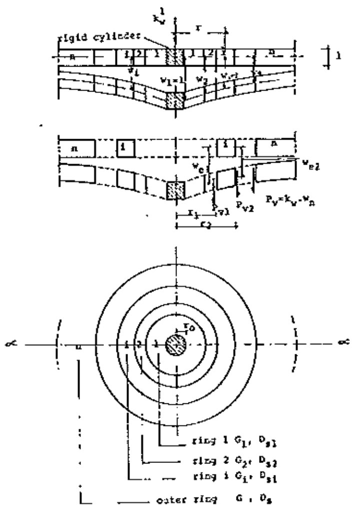

Fig. 1 Vertical Displacement of Cylinder in Horizontally Varying Soil.

linearity because the shear modulus and material damping of soil depend on the strain level which may vary along a considerable distance. Also, the omission of ring inertia forces will change the true behaviour of the system. Therefore, the present model includes the mass of soil rings and allows for as many rings as are considered desirable.

The decrease of shear modulus G and the increase of material damping towards the pile modify the complex soil reactions and stiffnesses of piles with frequency, and results in increasing the vibration amplitudes.

The pile is assumed vertical, linearly elastic and of circular cross section that vary at the interfaces of the layers. If the pile head lies above the soil surface, or if the pile is assumed to be separated from the soil, the adjacent layers are modelled as void (Fig. 2).

SOIL RESISTANCE

During vibration, the pile will receive a soil resistance of two types (1) reactions from the surrounding soils (dynamic soil reactions) which are distributed along the pile length, and (2) concentrated tip reactions from the soil below the pile tip. Dynamic soil reactions of homogeneous layers are given for all vibration modes in Novak et al. (1978) for the plane strain case. They have proved to be very useful in pile dynamics (El-Sharnouby & Novak 1985; Novak 1979; Novak & Aboul-ella 1978 a). Tip reactions are taken equal to those of a viscoelastic half-space as described by Novak & Aboul-ella (1978 a). If the soil properties vary, in each layer, with horizontal distance from the pile, the soil reactions can be derived following the pattern of the stiffness matrix method. The layer is divided into rings surrounding the rigid cylinder and the stiffness matrix of each ring is established. The overall stiffness matrix of the soil layer (of unit length in height) as a whole is assembled from the ring stiffness matrices. Hence, by applying the proper boundary conditions, the soil reactions are obtained.

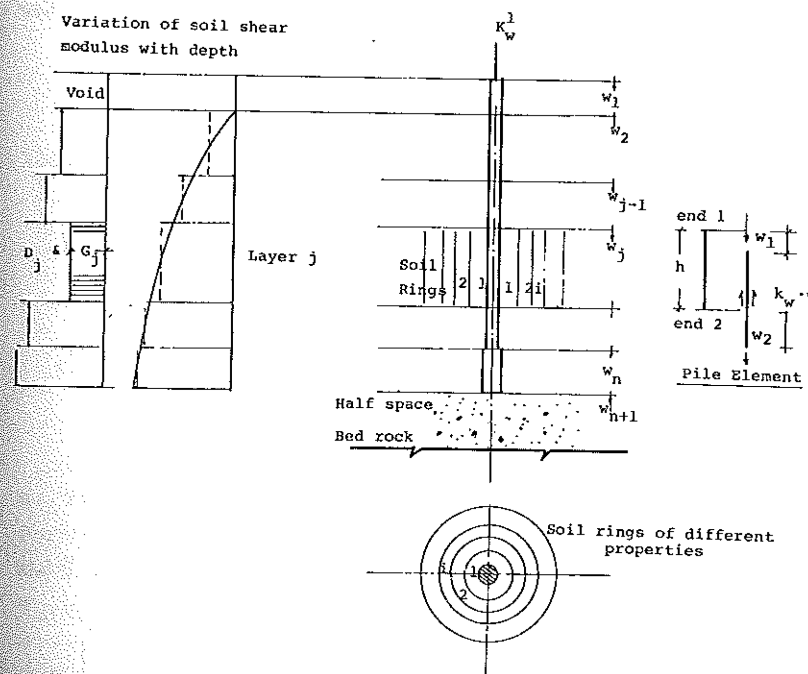

Fig. 2 Types of Piles Embedded in Layered Media and Vertical Vibration.

Ring Stiffness Matrix in Vertical Vibration

Consider only one ring of soil bounded by the radii r_1 and r_2 (Fig. 1). The stiffness matrix of this ring is the relationship between the two end shear forces P_{v1} and P_{v2} (Fig. 1) and the corresponding end displacements w_{e1} and w_{e2}. The amplitude of the vertical displacement w(r) of a homogeneous medium at radius r can be obtained from the general solution of the equation of motion of that medium. This amplitude is given by Novak et al. (1978) as :

$$w(r) = A\ K_0\ (sr) + B\ I_0\ (sr) \qquad (1)$$

in which

A, B = constants

K_0, I_0 = modified Bessel functions of order zero

$$sr = \frac{\omega\, r}{Vs_1\sqrt{1 + i\,Ds_1}}\ i$$

ω = circular frequency

r = radius

$i = \sqrt{-1}$

$Vs_1 = \sqrt{G_1 / \rho_1}$ = shear wave velocity of ring i

Ds_1 = material damping of ring i

G_1 = shear modulus of ring i

ρ_1 = mass density of ring i

For a ring with r_2 = infinity (i.e. outer region) the constant B in equation 1 must be equal to zero in order to have displacements decaying with horizontal distance.

The shear stress τ_r is

$$\tau_r = G_i\ (1 + iDs_1)\ \frac{\partial w\ (r)}{\partial r}$$

$$= -G_i\ (1 + iDs_i)\ s\ [A\ K_1\ (sr) + B\ I_1\ (sr)] \qquad (2)$$

Using the above shear stress, the end shear forces of the ring i (Fig. 1) can be given as :

$$P_{v1} = -\int_0^{2\pi} \tau_r\ (r = r_1)\ r_1\ d\theta$$

$$= 2\pi\ G_i\ (1 + iDs_i)\ sr_1\ [A\ K_1\ (sr_1) + B\ I_1\ (sr_1)]$$

and

$$P_{v2} = -2\pi\ G_i\ (1 + iDs_1)\ sr_2\ [A\ K_2\ (sr_2) + B\ I_1\ (sr_2)] \qquad (3)$$

The arguments sr_1 and sr_2 are complex dimensionless frequencies

$$a^*_1 = \frac{\omega\, r_1}{Vs_i\sqrt{1 + iDs_i}}\ i = \frac{a_{01}\ i}{\sqrt{1 + i\,Ds_i}} = sr_1$$

$$a^*_2 = \frac{\omega\, r_2}{Vs_i\sqrt{1 + iDs_i}}\ i = \frac{a_{02}\ i}{\sqrt{1 + i\,Ds_i}} = sr_2 \qquad (4)$$

in which the real dimensionless frequencies $a_{01} = \omega\, r_1/Vs_1$ and $a_{02} = \omega\, r_2/Vs_1$. Then the complex ring stiffnesses are defined by the boundary conditions $w(r_1) = w_{e1}$ and $w(r_2) = w_{e2}$. These conditions yield the constants A and B. The end shear forces P_{v1} and P_{v2} corresponding to end displacements w_{e1}, w_{e2} are :

$$\begin{bmatrix} P_{v1} \\ P_{v2} \end{bmatrix} = \begin{bmatrix} K_{R11} & K_{R12} \\ K_{R21} & K_{R22} \end{bmatrix} \begin{bmatrix} w_{e1} \\ w_{e2} \end{bmatrix} \qquad (5)$$

in which the elements $K_{R11} \dots K_{R22}$ of the ring stiffness matrix are :

$$K_{R11} = \Phi \, {}^*a_1 \left[\frac{K_1(a_1^*) \cdot I_0(a_2^*)}{K_0(a_1^*) \cdot I_0(a_1^*)} - \frac{I_1(a_1^*) K_0(a_2^*)}{I_0(a_1^*) K_0(a_1^*)} \right]$$

$$K_{R12} = \Phi \, {}^*a_1 \left[I_1(a_1^*) / I_0(a_1^*) - K_1(a_1^*) / K_0(a_1^*) \right]$$

$$K_{R21} = \Phi \, {}^*a_2 \left[\frac{I_1(a_2^*) \; K_0(a_2^*)}{I_0(a_1^*) \; K_0(a_1^*)} - \frac{K_1(a_2^*) \; I_0(a_2^*)}{K_0(a_1^*) \; I_0(a_1^*)} \right] \quad (6)$$

$$K_{R22} = \Phi \, {}^*a_2 \left[K_1(a_2^*) / K_0(a_1^*) - I_1(a_2^*) / I_0(a_1^*) \right]$$

in which (7)

$$\Phi = 2 \pi G_i (1 + i Ds_i) \left[I_0(a_2^*) / I_0(a_1^*) - K_0(a_2^*) / K_0(a_1^*) \right]$$

The stiffness of the outer media (infinite region) is the shear force that can produce a unit displacement amplitude of this region at $r = r_n$. Therefore, the stiffness of the outer media can be given as :

$$2 \pi G (1 + i Ds) a_0^* K_1(a_0^*) / K_0(a_0^*) \quad (8)$$

in which $a_0^* = \dfrac{a_0 \quad i}{\sqrt{1 + iDs}}$ and $a_0 = \dfrac{\omega r_n}{Vs}$

Soil Reaction of the Layer k_w

k_w is the shear force that is required at the interface between the rigid cylinder and the first ring in order to make w_1 equal unity. Using the stiffness matrices of all rings and the stiffness of the outer region, the stiffness matrix of the whole layer can be assembled. This matrix is the relationship between the external forces and displacements at the interfaces between rings. It can be partitioned and given as :

$$\begin{bmatrix} k_w \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} A & C^T \\ \hline C & H \end{array} \right] \begin{bmatrix} 1 \\ \hline \Delta \end{bmatrix} \quad (9)$$

External forces — stiffness matrix of the whole layer — displacements

Therefore, the soil reaction kw can be given as :

$$k_w = A - C^T H^{-1} C$$
$$= G \, [S_{w1} + i \, S_{w2}] \quad (10)$$

in which $G = G_n$, the shear modulus of the outer region and the dimensionless parameters $S_{w1,2}$ are real. The parameter S_{w1} represents the real stiffness of the soil layer of unit length and S_{w2} represents the damping.

In order to demonstrate the effect of variation of soil properties with distance an example has been analyzed with assumed values of G and Ds for each ring as shown in Fig. 3. $S_{w1,2}$ are given for the case of variable soil as well as the case of constant soil properties (homogeneous soil) using the same developed computer program. This has been done in order to check the program by comparing $S_{w1,2}$ of the homogeneous layer with those given in Novak et al. (1978).

It can be seen from Fig. 3 that the decrease of soil shear modulus G, accompanied by an increase of material damping Ds towards the pile, make parameters $S_{w1,2}$, and after that the pile's parameters, frequency dependent which can be easily incorporated in the analysis of pile-supported structures.

COMPLEX STIFFNESS OF PILE Kw^1

The pile is viewed as composed of prismatic elements extending between the interfaces of layers (Fig. 2). The stiffness matrix of each element is determined first using the soil reaction described by equation 10. This matrix is

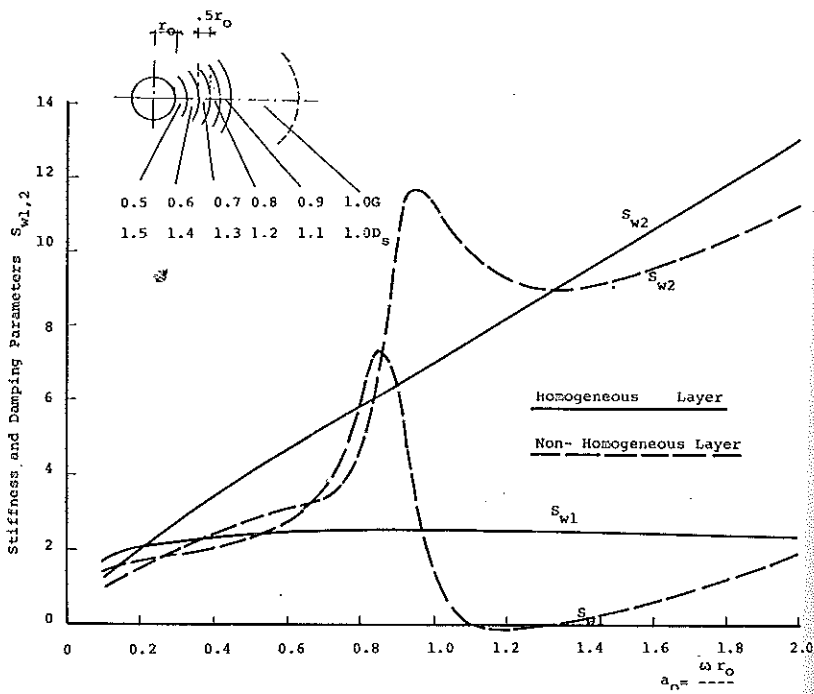

Fig. 3 Vertical Stiffness and Damping Parameter S (D = 0.1).

complex and includes the properties of both the pile and the soil. The overall stiffness matrix of the whole pile is assembled and solved in order to yield the vertical stiffness (impedence function) of the pile, K_w^1, which is the external force acting at the pile head in order to produce unit vibration amplitude of the head. A method of doing so is described in Novak & Aboul-ella (1978 a). It is convenient to split the complex stiffness, K_w^1, into its real part (true stiffness) and imaginary part (damping) and introduce the constant of equivalent viscous damping, C_w^1. Thus, the complex vertical stiffness of one pile is also given as :

$$K_w^1 = k_w^1 + i \omega c_w^1 \qquad (11)$$

Introducing the dimensionless stiffness and damping parameters f_{w1}, f_{w2}, the real stiffness of one pile

$$k_w^1 = \text{Real } K_w^1 = Ep.A_1.\ f_{w1} / r_1 \qquad (12)$$

and the constant of equivalent viscous damping

$$c_w^1 = \frac{1}{\omega} \text{ Imaginary } K_w^1 = Ep.A_1.f_{w2} / V_n \qquad (13)$$

In these equations A_1, r_1 = the area and radius of the topmost element of the pile, respectively, Ep = Young's modulus of piles, and V_n = shear wave velocity of soil in the lowest layer.

The dimensionless parameters $f_{w1,2}$ are suitable for parametric studies which are given in Novak & Aboul-ella (1978 b) for layered soil in which each layer is homogeneous. Here an example is given for the case of a soil having

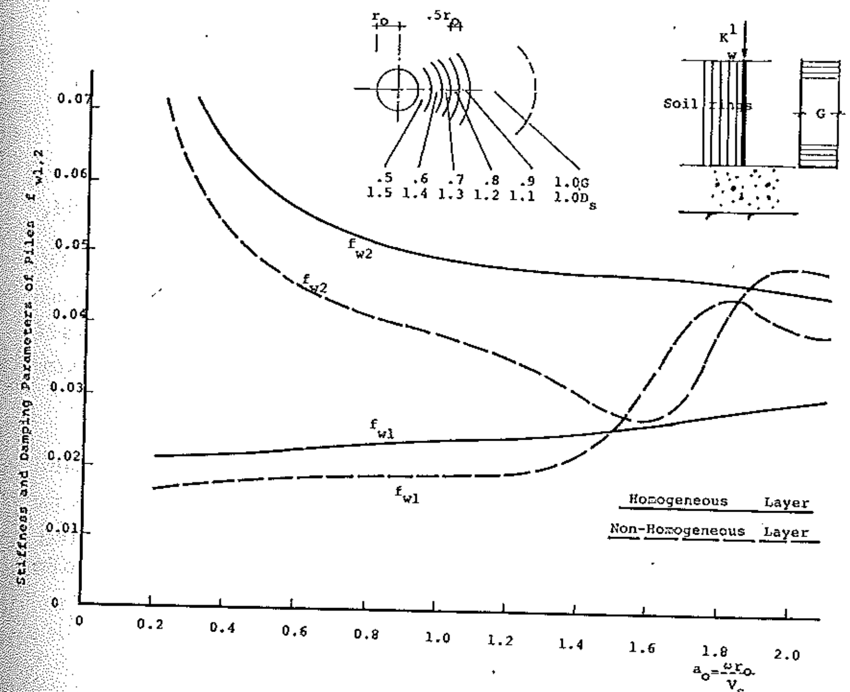

Fig. 4 Dimensionless Vertical Stiffness and Damping of Piles f (D = 0.1)

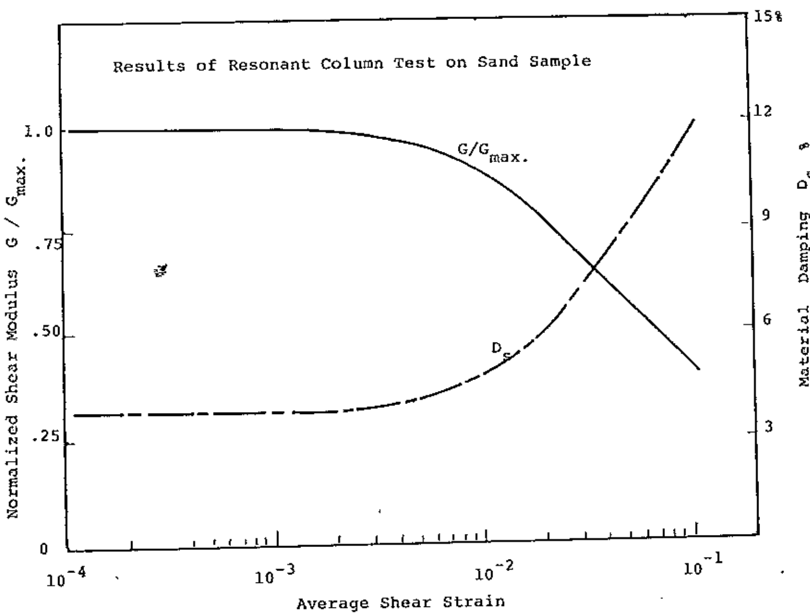

Fig. 5 Variation of Shear Modulus and Material Damping with Strain.

properties varying with distance. All data are given in Fig. 4 as well as f_{w12} for both cases i.e., homogeneous layer and horizontally varying soil layer. At small and moderate values of the dimensionless frequency a_0, which are the cases of pile supported foundations, the decrease of G and the increase of Ds towards the pile generally decrease the complex stiffness of the pile i.e. decrease $f_{w1,2}$.

The decrease of $f_{w1,2}$ will decrease the natural frequencies of pile-supported sturctures and increase their resonant amplitudes. In practical analysis the relationships between G, Ds and strains can be experimentally determined (Fig. 5). Therefore the adjusted values of G and Ds can be calculated according to the strain level at the centre of each ring.

RESPONSE OF BLOCK FOUNDATIONS

Piles in a group are connected by the soil and thus interact with one another.

This pile-soil-pile interaction modifies group stiffness and damping and can be studied by many approaches. El-Sharnouby (El-Sharnouby and Novak 1985) made use of the stiffness method while Wolf (Wolf & Arx 1978) has used the finite element technique. Novak (Novak & El-Sharnouby 1983) has recommended the use of dynamic interaction coefficiens which have been successfully used by Kaynia (Kaynia & Kausel 1982). The interaction factors, which are used by Poulos (1979; Polous & Mattes 1974) for static stiffnesses, are still approximate because they are calculated from the displacements of soil in which the piles are absent except for the reference pile.

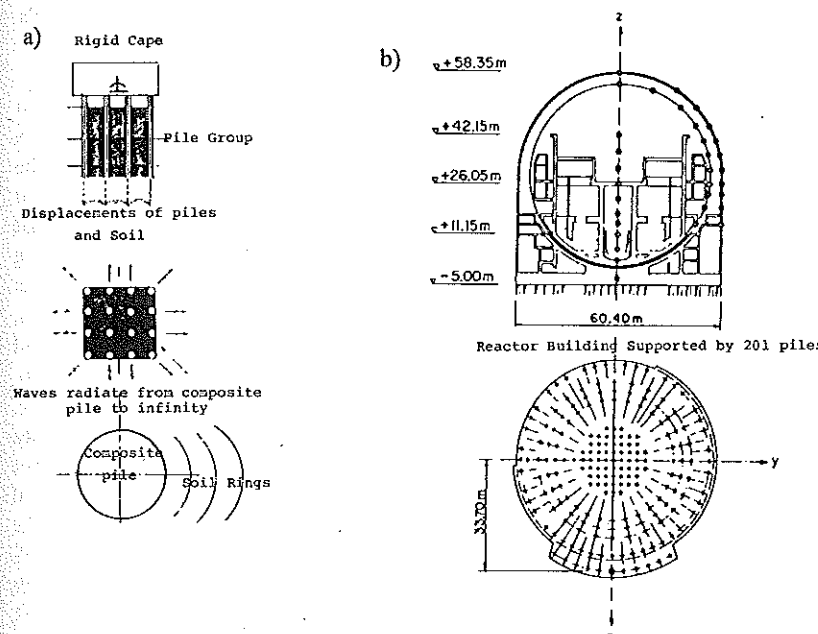

Fig. 6 Composite Pile Model for Pile Groups.

If the piles are closely spaced and connected by rigid caps (Fig. 6-a), the piles and the enclosed soil will vibrate as one body which receives soil reaction to its motion from the outside boundaries of the group. Therefore, the group of piles and the soil inside them can be considered, approximately, as one large composite pile with an effective area equal to the area of piles, plus the area of

Table 1 : Stiffness constants of 201 piles of Angra 2 Nuclear Power Plant

METHOD		REAL STIFFNESS kw[1]	
Present		432.7	GN/m
El-Sharnouby & Novak	1985	404.0	"
Wolf & Arx	1978	652.3	"

soil multiplied by the modular ratio Es/Ep, where Es and Ep is the Young's modulus for soil and piles respectively.

Aboul-ella (1984) had introduced the model of the composite pile in which generated geometric damping is calculated from waves that propagate from piles to infinity and, therefore, it eliminates the effect of wave scattering and the generation of standing waves within the group of piles. The model is simple but very versatile and has the advantage of using the available programs that calculate the impedance functions of one pile, with or without soil varying vertically and or horizontally, at negligible computing costs. These programs, as those developed by the author (1984) and Novak (Novak & Aboul-ella 1978a), are well prepared and can analyze continuous or stepwise variation of soil and pile properties with depth and an arbitrary relaxation of pile tips.

The model is approximate and therefore a comparison with other theories and experiments is desirable. The stiffness (real part of the complex stiffness) of 201 piles supporting the reactor building of Angra 2 nuclear power plant in Brazil (Fig 6-b) is calculated using the composite pile model and is compared with the results of the finite element method by Wolf and Arx (1978) and the stiffness method by El-Sharnouby and Novak (1985). The comparison (Table 1) indicates a full agreement with El-Sharnouby and a reasonable agreement with Wolf and, therefore it shows the usefullness of the composite pile model. The difference in stiffness between Wolf and the present approaches could be due to the very large size of the group, the different assumption made in the two approaches, the very irregular soil profile, and a very low pile-soil stiffness ratio. Also Wolf has lumped every four piles into one which could contribute to the difference in stiffness (El-Sharnouby & Novak (1985)).

Finally the experimental response curve measured for a small pile foundation is compared with that of the present composite pile (Fig. 7). All the data from

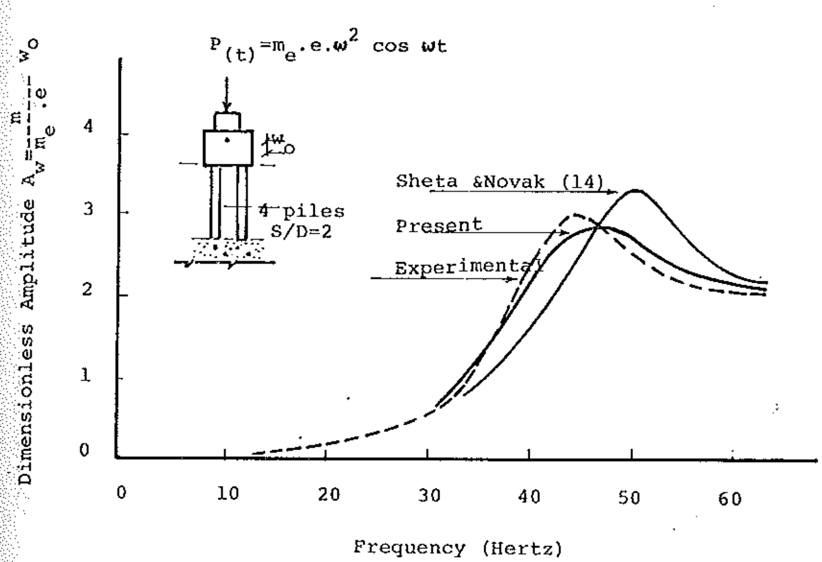

Fig. 7 Comparsion of present approach with Experiment.

this experiment were described by Sheta & Novak (1982). The comparison between the composite pile, the experiment, and the approach of Sheta supports the use of a composite pile model for closely spaced pile groups.

An example of the vertical response of a block machine foundation is shown in Fig. 8 in which all the input data are given. The increase of the resonant amplitude, due to the variations of G and Ds, is about 20% and depends mainly on the chosen values of G and Ds of the soil rings surrounding the piles i.e. if weaker rings are chosen, the increase of the resonant amplitude will become more than the one shown in Fig. 8

SUMMARY AND CONCLUSIONS.

A theory for vertical vibration of piles in horizontally varying soil is presented. This approach can be used to study the non-linear behaviour of piles and the effect of weak zones in regions of high stresses around piles. The solution is based on the matrix stiffness method for which the soil layer is divided into rings of different properties and the ring stiffness matrix is formulated. The decrease

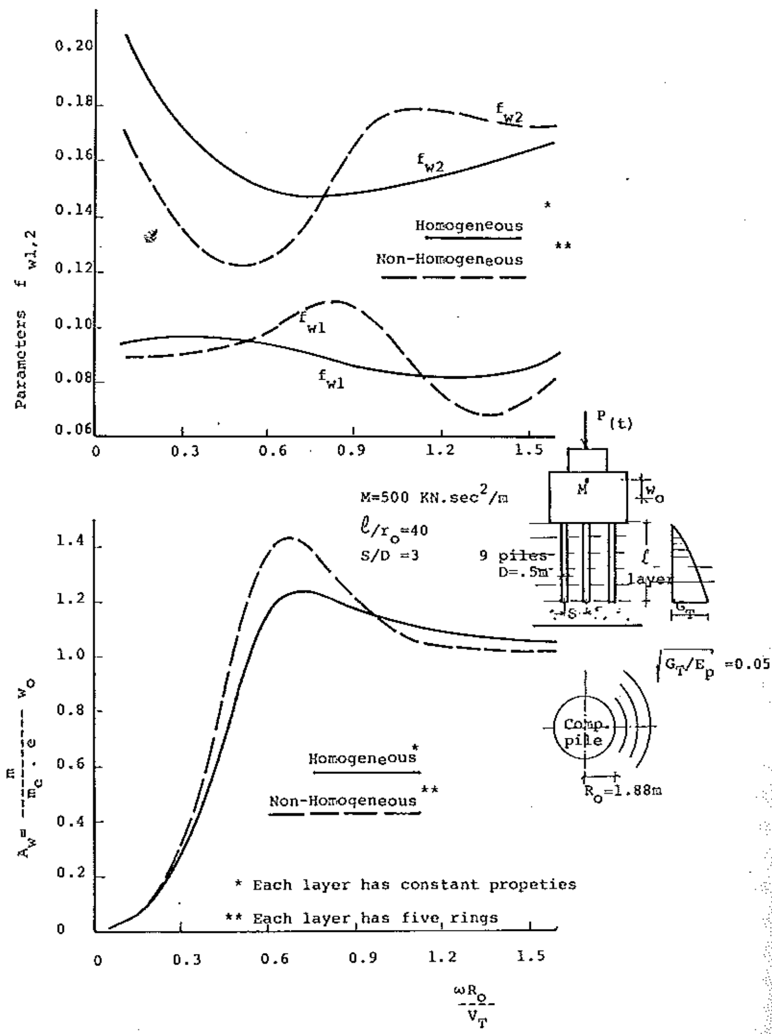

Fig. 8 Vertical Response of Block Foundation.

of shear modulus, accompanied by an increase of material damping of soil towards the pile, decreases the vibrational amplitudes of pile-supported foundations.

ACKNOWLEDGEMENT

The study is supported by a research grant from King Abdulaziz City for Science and Technology (KACST) which is gratefully acknowledged.

REFERENCES

ABOUL-ELLA, F. (1984). "Dynamic Behaviour of Pile - Supported Structures with Reference to Jubail Soil", *The Arabian Journal for Science and Engineering*, Vol. 9, No. 3, pp. 209-220.

ABOUL-ELLA, F., & NOVAK, M. (1980). "Dynamic Response of pile-supported Frame Foundations" *Journal of the Engineering Mechanics Division, ASCE*, Vol. 106, No. EM6.

EL-SHARNOUBY, B., & NOVAK, M. (1985). "Static and Low-Frequency Response of Pile groups" *Canadian Geotechnical Journal*, Vol. 22, No. 1, pp. 79-94.

HINDY, A., & NOVAK, M., (1980). "Earthquake Response of Buried Insulated Pipes", *Journal of Engineering Mechanics Div. ASCE*, Vol. 106 No. EM6.

KAYNIA, A.M., & KAUSEL, E. (1982). "Dynamic Behaviour of Pile Group", *2nd Int. Conf. on Numerical Methods in Offshore Piling*, Austin, Texas.

NOVAK, M., (1979). "Soil-pile Interaction Under Dynamic Loads" *Conference of Numerical Methods in Offshore piling*, Institution of Civil Engineering, London, England.

NOVAK, M., & ABOUL-ELLA, F., (1978 a). "Impedance Functions of Piles in Layered Media", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 104, No. EM3, Proc. Paper 13847, pp. 643-661.

NOVAK, M., & ABOUL-ELLA, F., (1978 b). "Stiffness and Damping of Piles in Layered Media", *Earthquake Engineering an Soil Dynamics*, Vol. II, ASCE, pp. 704-719.

NOVAK, M., NOGAMI, T., & ABOUL-ELLA, F., (1978). "Dynamic Soil Reactions for Plane Strain Case", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 104, No. EM4.

NOVAK, M., & EL-SHARNOUBY, B., (1983). "Stiffness Constants of Single Piles", *Journal of Geotechnical Engineering, ASCE*, Vol. 109, No. 7, pp. 961-974.

POULOS, H.G., (1979). "Group Factors for Pile-Deflection Estimation", *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 105, No. GT 12, Proceeding of Paper 15032, pp. 1489-1509.

POULOS, H.G., & MATTES, N.S., (1974). "Settlement of Pile Groups Bearing on Stiffer Strata", *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 100, No. GT 2, pp. 158-190.

SHETA, M., & NOVAK, M., (1980). "Approximate Approach to Contact Problems of Piles", *Research Report No. GEOT-7-80*, Faculty of Engineering Science, University of Western Ontariao.

SHETA, M., & NOVAK, M., (1982). "Vertical Vibration of Pile Groups", *The Journal of Geotechnical Engineering Division, ASCE*, Vol. 108, Vo. GT4, pp. 570-590.

WOLF, J.P., & ARX, G.A., (1978). "Impedance Functions of a group of Vertical Piles", *Earthquake Engineering and Soil Dynamics, Vol. II, ASCE*, pp. 1024-1041.

NOTATION

A_w = dimensionless vertical amplitude of footing

a_0 = r_0 w/Vs = dimensionless frequency

$C_w{}^I$ = damping constant of one pile

Ds = material damping constant of soil

Ep = Young's modulus of pile

Es = Young's modulus of soil

$f_{w1,2}$ = dimensionless vertical stiffness and damping parameters of pile

G = shear modulus of soil

G_T = shear modulus of soil at pile tip (lowest layer)

h = length of pile element

$I_{0,1}$ = modified Bessel functions of first kind of order zero and one, respectively.

I = moment of inertia of pile cross section

$K_w{}^I$ = complex stiffness of pile at head

$K_{0,1}$ = modified Bessel functions of second kind of order zero and one, respectively.

$k_w{}^I$ = real stiffness of one pile

ℓ = length of pile

m = mass of footing

r_0 = pile radius

$S_{w1,2}$ = dimensionless parameters of vertical soil resistance.

Vs = shear wave velocity of soil

W_0 = vertical amplitude of footing

τ = shear stress

∂ = mass density of soil

ω = circular frequency