

GEOTECHNICAL ENGINEERING

Journal of SOUTHEAST ASIAN SOCIETY OF SOIL ENGINEERING

Sponsored by ASIAN INSTITUTE OF TECHNOLOGY

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COUPLED SLIDING AND ROCKING OF FOUNDATIONS ON LAYERED MEDIA

RICHARD A. PARMELEE*, RAYMOND J. KRIZEK* and DINESH C. GUPTA+

SYNOPSIS

A two-dimensional, lumped-parameter model is used to investigate the steady-state coupled sliding and rocking behavior of an infinitely long, rigid foundation bonded to the surface of a linearly elastic, layered half-space. Wide ranges of soil properties, foundation dimensions, and thicknesses of the top layer are considered, and typical results are presented for the maximum amplitudes of the sliding and rocking oscillations and the frequencies at which these maximum amplitudes occur. The natural frequencies of the system with radiation damping are dependent on the shear wave velocity and Poisson's ratio of the half-space material, the geometric proportions of the foundation, and the thickness of the top layer. The total peak amplitude of vibration for the coupled motion is expressed in terms of a linear function whose coefficients are dependent on Poisson's ratio, the foundation dimensions, and the thickness of the top layer, and graphical relationships are presented to facilitate the determination of these coefficients. The results clearly illustrate that stratification of the soil may significantly affect the total peak amplitude of the foundation for a given exciting force, but this influence becomes negligible when the top layer is thicker than one and a half times the width of the foundation.

INTRODUCTION

The formulation of most dynamic soil-foundation interaction problems incorporates the assumption that the foundation rests on the surface of an idealized homogeneous, isotropic, linearly elastic half-space, and the variation of soil properties with depth is not taken into account. However, there are many practical situations in which the soil medium underlying the foundation may be stratified, and the occurrence of nearly parallel homogeneous layers with different properties is quite common. For such cases it is important to evaluate the influence of soil layering on the natural frequency and the amplitude. As long as amplitudes remain small (associated strains on the order of 10^{-5} or 10^{-6}), the assumption of linear elasticity has been used very successfully (RICHART et al, 1970; D'APPOLONIA, 1970) for such problems, and this assumption will be employed herein for each layer considered. Nevertheless,

^{*} Professor of Civil Engineering, The Technological Institute, Northwestern University, Evanston, Illinois, U.S.A.

⁺ Structural Engineering Analyst, Sargent & Lundy Engineers, Chicago, Illinois, U.S.A. *Discussion* on this paper is open until 1 November 1974.

despite the simplifying assumption of linear elasticity, the complexity of the dual integral equations which arise from the mixed boundary value problem has until a few years ago precluded any rigorous analytical solution to this problem. However, recent advances in the development of appropriate mathematical models (AGABEIN et al, 1968) have enabled the steady-state, coupled response of harmonically excited, infinitely long or circular, rigid foundations to be analyzed.

DESCRIPTION OF PROBLEM

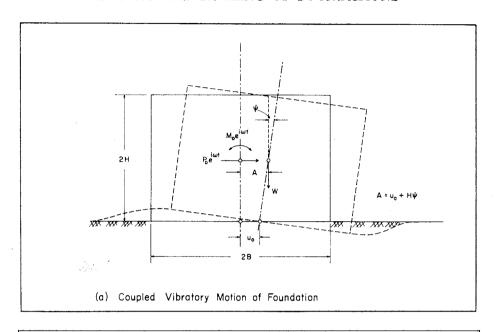
The problem treated herein consists of determining the coupled sliding and rocking response (natural frequency and amplitude) of an infinitely long, rigid foundation whose base is contiguous with the surface of a two-layered elastic medium; each layer is homogeneous and isotropic, and the interface is horizontal and parallel to the surface on which the foundation rests. The thickness of the top layer is D, whereas the bottom layer is semi-infinite, and the shear wave velocities and values of Poisson's ratio for the top and bottom layers are designated as V_{s1} , V_{s2} , v_1 , and v_2 , respectively. As illustrated in Fig. 1, the foundation has a width of 2B and a height of 2H, and it is assumed to be excited by a horizontally oscillating harmonic force of frequency ω and amplitude P_o acting at a distance H_T above its top.

BRIEF BACKGROUND

Axially Symmetric Solutions

In some of the earlier theoretical work related to this general subject, BYCROFT (1956) presented analytical solutions for the uncoupled vertical, torsional, sliding, and rocking response of a rigid circular plate on the surface of a homogeneous, isotropic, linearly elastic half-space. When coupled rocking and sliding oscillations of a rigid circular plate are considered, coupled pairs of dual integral equations result.

By a reorganization of the basic differential equations for the vibratory motion of a weightless, rigid, circular plate on the surface of an elastic half-space, HSIEH (1962) demonstrated that all modes of vibration could be represented by the equations of motion for the forced vibration of a lumped parameter (mass-spring-dashpot) system, in which the damping and spring factors are functions of the frequency of oscillation. He included in this study a description of these functions for sliding and rocking oscillations, and he illustrated their use in establishing the system of equations for simultaneous oscillations. Subsequently, HALL (1967) idealized the Bycroft solutions for



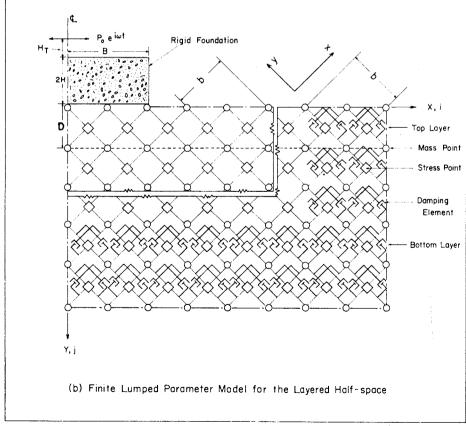


Fig. 1. Schematic description of problem and associated model.

pure rocking and pure sliding to develop the lumped parameter analogs for pure rocking and pure sliding oscillations, and he then used the stiffness factors and damping coefficients from these analogs to establish and solve approximately the equations for the simultaneous rocking and sliding motion of a circular plate on a homogeneous, isotropic, linearly elastic half-space. Using a somewhat similar approach, RATAY (1971) developed a solution for the simultaneous rocking and sliding motion of a rigid mass with a circular base; however, this solution is limited to soils with a Poisson's ratio of zero. Because of the single-mode-of-vibration assumption contained in their development, the adaptation of these elasticity solutions to the case of vibratory machine foundations has been limited to studies of the simultaneous, but uncoupled, motion of the foundation (MCNEILL, 1969).

An extension of these analyses to solve the general problem of the dynamic behavior of foundations on layered media or on an elastic medium whose stiffness varies with depth has not yet been accomplished. The available solutions indicate only qualitative trends for the dynamic response of rigid, axially symmetric foundations which rest on a single elastic layer; in addition, the boundary conditions and dynamic loading conditions are usually quite restrictive. For example, there have been attempts to solve the problem of a foundation resting on the surface of a homogeneous, isotropic, linearly elastic layer of infinite horizontal extent and underlain at a relatively shallow depth by a rigid boundary. Reissner (1937) outlined the method of solutions for the case of torsional oscillation of a circular foundation on a layered medium, and ARNOLD et al (1955) and Bycroft (1956) presented solutions for this problem. The vertical oscillation of a rigid circular foundation on an elastic layer underlain by a rigid boundary was treated by ARNOLD et al (1955), BYCROFT (1956) and WARBURTON (1957). Thus far, no attempt has been reported to address the problem of simultaneous or coupled sliding and rocking oscillations of circular rigid foundations on a layered medium, even for restrictive boundary conditions.

Plane Strain Solutions

KARASUDHI et al (1968) superimposed the results of pure sliding and pure rocking vibrations to determine an approximate solution for the coupled sliding and rocking response of an infinitely long rigid foundation on the surface of an elastic half-space. Although, their solution is approximate in nature, the results clearly demonstrate that coupling effects can be quite significant, and it is reasonable to expect that these effects will also be important for the coupled response of foundations resting on layered media. Gupta et al (1973) applied a numerical approach developed by Agabein et al

(1968) to the problem of coupled harmonic sliding and rocking of an infinitely long, rigid foundation bonded to the surface of an elastic half-space, but no solution has thus far been reported for the case where soil stratification occurs.

DESCRIPTION OF MODEL

Historical Background

The lumped-parameter model used in this study to simulate the soil layers was originally developed by Ang & Harper (1964) to evaluate numerically the states of stress and strain in a continuous solid wherein certain finite regions have been strained beyond their elastic limits. Höeg et al (1968) used essentially this same model to study the factors affecting the magnitude of the settlement and the nature of the load-settlement curve for statically loaded strip foundations resting on an idealized elastic-plastic soil. Höeg & RAO (1970) introduced nonlinear spring connections between the lumped masses and studied the response of strip foundations to impulse loads. AGABEIN et al (1968) modified the Ang and Harper model to include finite boundaries and boundary damping elements to account for the effect of radiation damping in a semi-infinite medium, thereby developing a fairly general twodimensional model for the study of seismic soil-structure interaction problems. This discrete element model yields results which are in satisfactory agreement with the solution for coupled motion developed by Karasudhi et al (1968), and an extended version of this basic model is employed in this investigation.

Nature of Model and Boundary Conditions

The extended lumped-parameter model used in this study is shown in Fig. 1 b and consists of a finite number of mass points (five of which are assumed to exist at the soil-foundation interface) and stress points. The requirement that the response of the model be essentially the same as that of the semi-infinite layered medium is accomplished by imposing the following conditions at the boundaries:

- (a) the displacements of all mass points in contact with the base of the rigid foundation are the same as those of the corresponding points of the foundation itself; that is, the foundation is rigid and is bonded to the soil,
- (b) the free surfaces beyond the sides of the rigid foundation are stress free,
- (c) the displacements of the mass points along the side and bottom boundaries of the model are zero, and
- (d) the elastic layers are contiguous at their interface.

Size of Model

The dimensions of the original model developed by AGABEIN et al (1968) were 4B in the X-direction and 3.5B in the Y-direction, where B is the half-width of the foundation; this model size yields an accuracy of approximately five per cent and requires reasonable computational times. In the extended model for the behavior of a foundation on a stratified medium, the size of the model is 4B in the X-direction and 4.5B in the Y-direction. This larger model provides better accuracy and facilitates the study of foundation response for a D/B range from zero to 3.0. The computational time required for use of this extended model increased only slightly.

Boundary Damping

Viscous damping elements have been provided along the side boundaries of the top layer and along the bottom and side boundaries of the bottom layer of the finite model in order to simulate radiation damping. The damping coefficients for these elements are a function of the density, shear wave velocity, and Poisson's ratio for the layer under consideration. The effects and criterion for choosing the viscous damping elements have been described by AGABEIN et al (1968).

Equations of Motion for Interior Mass Points

A typical interior mass point contains the mass $m_o = \frac{1}{2} \rho b^2$, where ρ is the average mass density of the layer and b is the mesh size of the model. The masses of the surface mass points and the mass points in contact with the boundary of the foundation are proportionately adjusted. The displacements, velocities, and accelerations are defined at the mass points, while the average stresses and strains are defined at the stress points, which are assumed to be in a state of homogeneous stress and strain. The strains at a stress point are determined from the displacements of adjacent mass points, and the forces acting on a mass point are calculated from the stresses at adjacent stress points. As explained in detail by AGABEIN et al (1968), the equations of motion for a typical interior mass point (i, j) in terms of the displacements u and v, in the x and y directions, respectively, are:

$$\frac{G}{b^{2}} \left[u(i+2,j+2) + u(i-2,j+2) - 4u(i,j) + u(i-2,j-2) + u(i+2,j-2) \right]
+ \frac{\lambda + G}{b^{2}} \left[u(i+2,j+2) - 2u(i,j) + u(i-2,j-2) - v(i-2,j) + v(i,j-2) - v(i+2,j) + v(i,j+2) \right] = \rho \ddot{u}(i,j) \quad ... \quad ...$$

and

$$\frac{G}{b^{2}} \left[v(i-2,j+2) + v(i-2,j-2) - 4v(i,j) + v(i+2,j-2) + v(i+2,j+2) \right]
+ \frac{\lambda + G}{b^{2}} \left[v(i-2,j+2) - 2v(i,j) + v(i+2,j-2) + u(i,j+2) - u(i-2,j) \right]
+ u(i,j-2) - u(i+2,j) = \rho \ddot{v}(i,j) (1b)$$

in which $\lambda = 2\nu G/(1-2\nu)$, and G and ν are the shear modulus and Poisson's ratio, respectively, for the layer under consideration.

Equations of Motion for Mass Points on Layer Interface

The stresses and strains at the stress points on either side of the layer interface are related due to the continuity of all stress and displacement components across the boundary; accordingly, the equations of motion for a typical interface mass point (i, j) in terms of the displacements u and v are:

$$\left(\frac{\lambda_2 + 2G_2}{b^2}\right) u(i+2, j+2) + \left(\frac{\lambda_2 + G_2}{b^2}\right) v(i, j+2) + \frac{G_2}{b^2} u(i-2, j+2) - \left(\frac{\lambda_2 + G_1}{b^2}\right) v(i+2, j) - \left(\frac{\lambda_1 + \lambda_2 + 3G_1 + 3G_2}{b^2}\right) u(i, j) - \left(\frac{\lambda_1 + G_2}{b^2}\right) v(i-2, j) + \frac{G_1}{b^2} u(i+2, j) + \left(\frac{\lambda_1 + G_1}{b^2}\right) v(i, j-2) + \left(\frac{\lambda_1 + 2G_1}{b^2}\right) u(i-2, j-2) = \rho \ddot{u}(i, j) \quad . \quad . \quad (2a)$$
and:

$$\left(\frac{G_2}{b^2}\right) v(i+2,j+2) + \left(\frac{\lambda_1 + G_2}{b^2}\right) u(i,j+2) + \left(\frac{\lambda_1 + 2G_1}{b^2}\right) v(i-2,j+2) - \left(\frac{\lambda_2 + G_2}{b^2}\right) u(i+2,j) + \left(\frac{\lambda_1 + \lambda_2 + 3G_1 + 3G_2}{b^2}\right) v(i,j) - \left(\frac{\lambda_1 + G_1}{b^2}\right) u(i-2,j) + \left(\frac{\lambda_2 + 2G_2}{b^2}\right) v(i+2,j-2) + \left(\frac{G_1 + \lambda_2}{b^2}\right) u(i,j-2) + \left(\frac{G_1}{b^2}\right) v(i-2,j-2) = \rho \ddot{v}(i,j) \quad . \quad (2b)$$

in which $\lambda_1 = 2v_1G_1/(1-2v_1)$ and $\lambda_2 = 2v_2G_2/(1-2v_2)$, G_1 and v_1 are the shear modulus and Poisson's ratio, respectively, for the top layer, and G_2 and v_2 are the shear modulus and Poisson's ratio, respectively, for the bottom layer. On the free surface the applied stresses are defined at fictitious stress points. For mass points (0, j) along the plane of symmetry, only motions normal to the plane are permissible, that is, v(0, j) = -u(0, j). For the mass points adjacent to the side and bottom boundaries, the equations of motion can be written as Eqs. 1a and 1b, but suitably modified to account for the damping elements placed in parallel with the stress points. The equations of motion

for the foundation mass involve the summation of all horizontal forces acting at the level of the foundation base and the summation of all moments about the center of gravity of the foundation mass. For the model described, there are 140 equations of equilibrium (or degrees of freedom)—128 for the 64 interior mass points (including 16 equations for the interface mass points), 10 for the surface mass points, and 2 for the foundation mass.

METHOD OF SOLUTION

The results of prime interest are (a) the total maximum amplitudes of the sliding and rocking oscillations and (b) the frequencies at which such maximum amplitudes occur. Soil-foundation systems may exhibit several natural frequencies, which correspond to the various number of degrees of freedom, and the lowest natural frequency is assumed herein to be the one of utmost practical significance. Since the general method of solution used to calculate these parameters has previously been explained in detail by AGABEIN et al (1968), only a brief description will be given herein.

The equations of motion for the system may be written as:

$$[M] \left\{ \ddot{s} \right\} + [C] \left\{ \dot{s} \right\} + [K] \left\{ s \right\} = \left\{ F \right\} e^{i\omega t} \dots (3)$$

where [M] is the mass matrix, [C] is the damping matrix, [K] is the stiffness matrix, $\{s\}$ is the displacement vector, and $\{F\}$ is the exciting force vector. Using the method of complex response, where:

$$\left\{ S \right\} = \left\{ S \right\} e^{i\omega t} ; \qquad S = S_1 + iS_2 \ldots \ldots (4)$$

we may write the equations of motion for the system as:

where:

$$[R] = -\omega^2[M] + i\omega[C] + [K]$$
 (6)

The solution to Eq. 5 was obtained by using the partitioning method (PRZE-MIENIECKI, 1968) of matrix multiplication. The response of the foundation was obtained for several frequencies of excitation by a force of constant amplitude, and the lowest resonant frequency corresponding to the peak response of the foundation was determined to an accuracy of 0.01 cps.

PARAMETER STUDY

To obtain an appreciation for the influence of the various independent variables on the amplitudes of motion and the associated damped (with radiation damping) natural frequencies, a representative parameter study was un-

dertaken, and the results are given in Figs. 2 through 5. Although the particular ranges for each variable are very selective, the overall study summarized in Table 1 is considered to encompass a broad variety of commonly encountered situations. A schematic diagram of the coupled rotation, Ψ , and the horizontal translation, u_0 , for the lowest mode of vibration of the rigid foundation is given in Fig. 1a. Interface damping between the foundation and the soil is neglected, and the influence of various foundation and soil parameters on the dynamic response of the system is examined for the case where the foundation is subjected to a steady-state harmonic excitation. A unit length of foundation normal to the direction of vibration is considered, and other limitations of the parameter study are explained in the notes at the bottom of Table 1.

Table 1. Scope of parameter study

Figure	v_1	v_2	$V_{\rm s1}/V_{\rm s2}$	H/B
2	0.35	0.35	0.25 0.50 1.00 2.00	0.50
3	0.25 0.25 0.45 0.45	0.25 0.45 0.25 0.45	1.00	0.50
4	0.25	0.25 0.35 0.45	0.50	0.50
	0.35	0.25 0.35 0.45	0.50	0.50
	0.45	0.25 0.35 0.45	0.50	0.50
5	0.35	0.35	0.50	0.25 0.50 1.00

Notes: 1. In all cases reported herein, V_{s2} is assumed to be 1000 ft/sec; for other values of V_{s2} , the results can be adjusted proportionately.

^{2.} The unit weights of the soil and foundation material are taken as 110 and 150 lb/ft³, respectively.

^{3.} The base width of the foundation is assumed to be 20 ft.

Natural Frequencies

The top parts of Figs. 2 through 5 illustrate the different relationships for the damped (with radiation damping) natural frequency, $f_{\rm nd}$, as a function of the layer thickness parameter, D/B, for various values of the shear wave velocity ratio, $V_{\rm s1}$ $V_{\rm s2}$, the foundation height-to-width ratio, H/B, and $v_{\rm 1}$ and $v_{\rm 2}$. Since $V_{\rm s2}$ is assumed equal to 1000 ft/sec in these figures, $f_{\rm nd}$ values determined therefrom must be multiplied by $V_{\rm s2}/1000$ (where $V_{\rm s2}$ is in ft/sec) if the bottom layer has a shear wave velocity other than 1000 ft/sec. Natural frequencies are not affected by the magnitude of the forcing function, as long as it remains small enough so that the assumptions of linear elasticity are not violated, and the variations observed in Figs. 2 through 5 are due to changes in the independent variables.

The top part of Fig. 2 illustrates the effect of $V_{\rm s1}/V_{\rm s2}$ and D/B on $f_{\rm nd}$ for H/B equal to 0.50 and v_1 and v_2 equal to 0.35. The natural frequency of the system is seen to increase as D/B increases for $V_{\rm s1}/V_{\rm s2}$ values greater than one, whereas it decreases as D/B increases for $V_{\rm s1}/V_{\rm s2}$ values less than one. For all practical purposes, if the top layer is as deep as the width of the foundation (that is, D/B=2), the natural frequency of the system is essentially the same as that which would be obtained if the top layer were infinitely thick.

The effects of variations in v_1 and v_2 on $f_{\rm nd}$ for $V_{\rm sl}/V_{\rm s2}$ equal to 1.00 and H/B equal to 0.50 are shown in the top part of Fig. 3. The natural frequency is seen to be weakly dependent on Poisson's ratio over a wide practical range. If v_1 is greater than v_2 , $f_{\rm nd}$ increases with increasing values of D/B, but the reverse is true if v_1 is less than v_2 ; in either case the response is essentially equivalent to that for an infinitely thick top layer if the top layer is slightly thicker than the width of the foundation.

The top parts of Fig. 4 show the effect of variations in v_2 and D/B on $f_{\rm nd}$ for various values of v_1 and for constant values of $V_{\rm s1}/V_{\rm s2}$ and H/B. As in the previous cases, the effect of layering essentially vanishes for D/B values greater than 2; for D/B less than 2, the predominant effect is due to the difference in the shear wave velocity of the two layers, but the influence of various combinations of v_1 and v_2 may be substantial, particularly for a very thin top layer. A comparison of values in Figs. 2 and 4 shows that, for D/B equal to 0.5, doubling or halving the shear wave velocity in the top layer may cause a change of 2 or 3 cps in $f_{\rm nd}$, and similarly, for a constant ratio of shear wave velocities, particular combinations of v_1 and v_2 may alter $f_{\rm nd}$ by 2 or 3 cps.

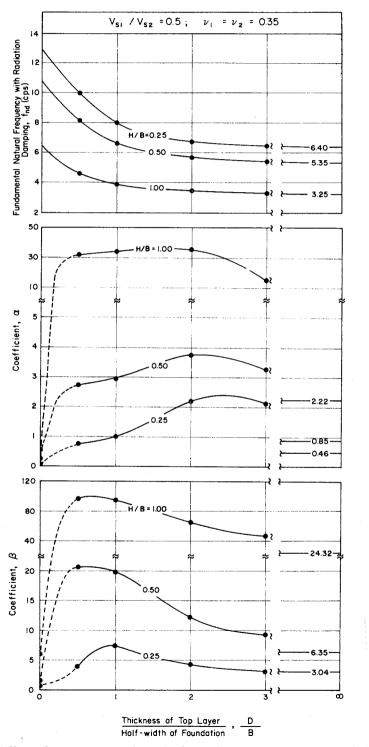


Fig. 2. Effect of wave propagation velocity ratio on response characteristics.

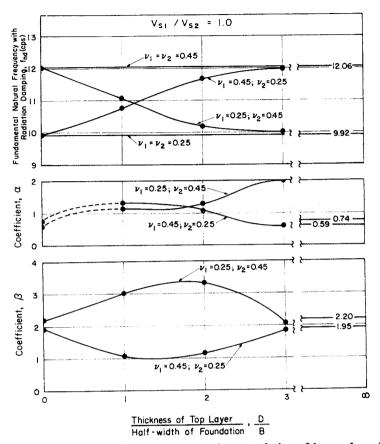


Fig. 3. Effect of Poisson's ratio on response characteristics of layered medium with homogeneous wave propagation velocity.

The influence of H/B on $f_{\rm nd}$ is illustrated in the top part of Fig. 5 as a function of D/B for $V_{\rm s1}/V_{\rm s2}$ equal to 0.50 and $v_{\rm 1}$ and $v_{\rm 2}$ both equal to 0.35 (the same as in Fig. 2). The fundamental natural frequency is seen to increase substantially as the height-to-width ratio of the foundation decreases; for example, if D/B equals zero, $f_{\rm nd}$ for H/B equal to 0.25 is on the order of 1.2 times that for H/B equal to 0.50 and twice that for H/B equal to 1.00. For all values of H/B, $f_{\rm nd}$ decreases with increasing values of D/B, and the effect of layering essentially disappears when D/B is greater than 2.

Amplitudes

The total peak amplitude, A, of the foundation motion is found to vary directly as the magnitude of the applied force, $P_{\rm o}$, and inversely as the square of the shear wave velocity of the bottom layer, $V_{\rm s2}$, as long as elastic theory is not violated. A representative parameter study indicates that this total peak response can be expressed as

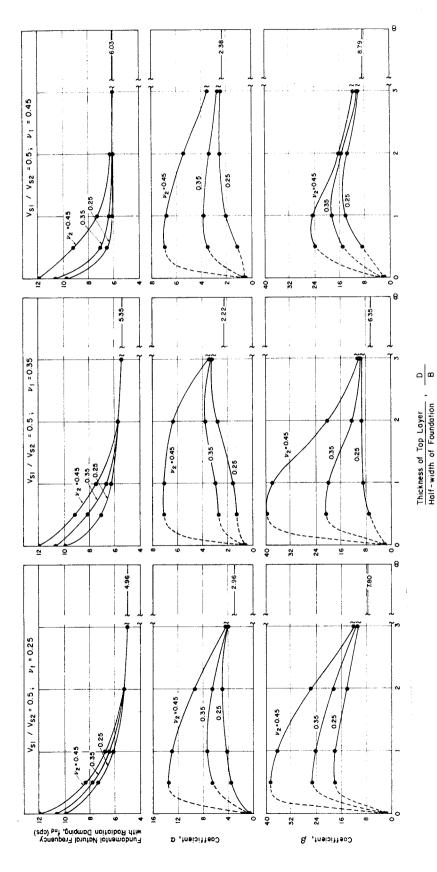


Fig. 4. Effect of Poisson's ratio on response characteristics.

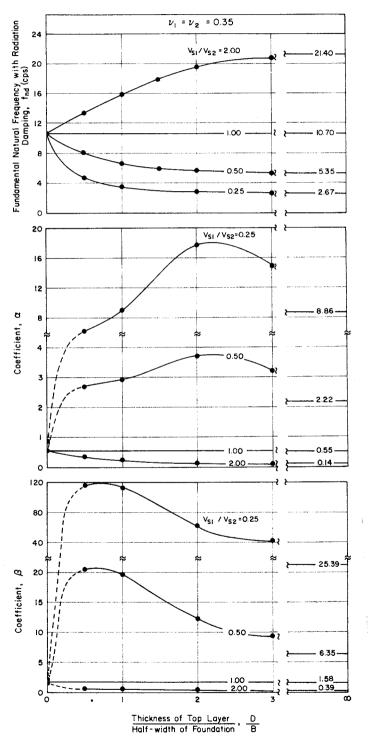


Fig. 5. Effect of foundation geometry on response characteristics.

$$A = \frac{P_{o}}{V_{c2}^{2}} \left[\alpha \left(\frac{H_{T}}{B} \right) + \beta \right] \dots (7)$$

in which A is in feet, P_0 is in pounds, V_{s2} is in feet per second, and α and β are coefficients which are plotted in the middle and lower parts, respectively, of Figs. 2 through 5. As can be seen from the values of these coefficients used in conjunction with Eq. 7, the peak amplitude increases significantly with increasing values of H/B (see Fig. 5) and decreasing values of $V_{\rm s1}/V_{\rm s2}$ (see Fig. 2); this is particularly evident because a change in scale is used to plot α and β in these two figures. In general, the effects of variations in the ratio of the shear wave velocities are much greater than the effects due to extreme variations in the values of Poisson's ratio for the two layers. However, the influence of v_1 and v_2 is also quite important in many cases and cannot be overlooked. For example, although values of the coefficients α and β in Fig. 3 are relatively small, differences of 100% and 200% are observed for various combinations of v_1 and v_2 at particular values of D/B. In Fig. 4 the effect of layering is seen to essentially disappear as D/B approaches 3; however, v_1 and v_2 do exert a substantial influence on values of α and β , and there are differences of several hundred per cent for D/B values around unity. Also, the amplitude response does not vary monotonically between the two limiting cases, but peak amplitudes for certain combinations of variables may increase by an order of magnitude over those for one or both limiting cases. In most situations involving stratification, the presence of a thin layer (D/B) less than 0.5) causes a rapid change in the amplitude response, although quantitative values for such response are usually not given herein for two reasons; first, computer costs preclude the determination of the many permutations needed to adequately define these portions of the curves, and second, the practical usefulness of this information is very limited, even if it were available. Finally, it is important to realize that the foregoing discussion and the results illustrated in Fig. 2 through 5 are concerned only with amplitudes at resonacne and do not necessarily represent trends at other frequencies.

CONCLUSIONS

Based on the use of a finite, lumped-parameter model to conduct a limited parameter study of the steady-state, coupled sliding and rocking response of an infinitely long, rigid foundation which is contiguous with the surface of a layered, elastic half-space and subjected to harmonic excitations, the following conclusions can be advanced:

- (1) The effect of layering on the damped (with radiation damping only) natural frequency of the system is essentially eliminated if the top layer is thicker than the width of the foundation, and the response is nearly the same as that obtained for the case where the top layer is infinitely thick.
- (2) For the case where the thickness of the top layer is equal to the width of the foundation, doubling or halving the shear wave velocity in the top layer, while maintaining Poisson's ratio constant for both layers, will essentially double or halve the natural frequency of the system, whereas reasonable variations in the values of Poisson's ratio for the two layers, while keeping the shear wave velocity constant for both layers, change the natural frequency by only 1 or 2 cps.
- (3) For the case where the thickness of the top layer is up to one-half the width of the foundation and where the shear wave velocity of the top layer is one-half that of the bottom layer, reasonable variations in the values of Poisson's ratio for the two layers cause a maximum change of 2 or 3 cps in the natural frequency of the system.
- (4) The geometrical configuration of the foundation exerts a significant influence on the natural frequency of the system; for a given foundation geometry and constant properties of the layered system, the natural frequency decreases as the thickness of the top layer increases, and it can double as the height-to-width ratio of the foundation decreases from unity to one-fourth.
- (5) The total peak amplitude of the coupled motion can be expressed as a linear function of the parameter which characterizes the point at which the exciting force is applied, and it is proportional to the magnitude of the applied force and inversely proportional to the square of the shear wave velocity of the bottom layer.
- (6) The presence of a thin top layer, which may have a thickness less than one-fourth the width of the foundation, causes a rapid change in the amplitude response in practically all cases considered.
- (7) The shear wave velocity of the top layer exerts a strong influence on the peak amplitude, which may increase by more than one order of magnitude over the homogeneous case if the shear wave velocity of the top layer is one-fourth that of the bottom layer.
- (8) Particular combinations of Poisson's ratio for the two layers may alter the peak amplitude by a factor of two or three, if the shear wave velocities of both layers are the same.

- (9) For cases where both the shear wave velocity and Poisson's ratio for each layer are different, the amplitude response does not vary monotonically between the two limiting cases; rather, the peak amplitude for certain combinations of variables may increase by an order of magnitude over that for one or both limiting cases.
- (10) The height-to-width ratio of the foundation may change the amplitude by more than an order of magnitude.
- (11) As far as amplitude response is concerned, the effect of layering does not essentially disappear until the thickness of the top layer is about one and one-half times the width of the foundation.

ACKNOWLEDGMENT

The research upon which this paper is based was supported in part by the National Science Foundation under Grant GK-1910 to Northwestern University. The computer time used for this study was furnished by the Vogelback Computing Center, and this support is gratefully appreciated.

APPENDIX: NOTATION

```
total peak amplitude of horizontal movement of the foundation;
\boldsymbol{A}
\boldsymbol{B}
            half-width of the strip foundation;
b
            mesh size of the model;
            damping matrix;
[C]
            thickness of the top layer;
D
            base of the natural logarithm;
e
            force vector;
            natural frequency of the system with radiation damping;
f_{\sf nd}
            \rho V_s^2; shear modulus;
G
            shear modulus of the top layer;
G_1
            shear modulus of the bottom layer;
G_2
            acceleration due to gravity;
g
        =
            half-height of the rigid foundation;
H
            height of applied force above the top of the foundation;
H_{\mathbf{T}}
            unit length in X-direction; also \sqrt{-1}:
i
            unit length of Y-direction;
            stiffness matrix:
[K]
        =
            mass matrix;
[M]
        =
            mass of typical interior mass point;
m_{\rm o}
            magnitude of the horizontal exciting force;
P_{\rm o}
```

[R] = modified complex stiffness matrix;

S = magnitude of the complex response;

 S_1 , S_2 = real and imaginary parts of S;

 $\{s\}$ = generalized displacement vector consisting of u and v components;

t = time;

 u, \ddot{u} = displacement and acceleration of mass elements in the model along the x-axis:

 V_s = shear wave velocity of the layer under consideration;

 $V_{\rm s1}$ = shear wave velocity of the top layer;

 $V_{\rm s2}$ = shear wave velocity of the bottom layer;

 v, \ddot{v} = displacement and accerleration of mass elements in the model along the y-axis;

X, Y = horizontal and vertical coordinate axes;

 $x, y = \text{coordinate axes at } 45^{\circ} \text{ to the } X, Y \text{ axes};$

 α , β = dimensionless coefficients;

 λ = Lamé's constant;

 λ_1 , λ_2 = Lamé's constant for top and bottom layer respectively;

υ = Poisson's ratio of the layer under consideration;

 v_1 = Poisson's ratio of the top layer;

 v_2 = Poisson's ratio of the bottom layer;

 ρ = Mass density;

 Ψ = rotational displacement of the foundation;

 ω = circular frequency.

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A CRITICAL STUDY OF THE UNIQUENESS OF STATE BOUNDARY SURFACE FOR SATURATED SPECIMENS OF KAOLIN

A.S. BALASUBRAMANIAM*

SYNOPSIS

This paper is concerned with a detailed study of the uniqueness of state boundary surface for saturated specimens of Kaolin. Specimens of Kaolin prepared from a slurry were sheared from several isotropic stress states under a wide variety of imposed stress paths in a conventional axisymmetric triaxial apparatus under stress controlled conditions. The data corresponding to the state paths followed by over 25 test specimens are presented and discussed. The effects of several other factors such as the initial one dimensional consolidation stress used in sample preparation, the load increment size, the load increment duration etc. on the state boundary surface are also considered in detail.

INTRODUCTION

The state boundary surface relating the voids ratio, e, the mean normal stress, p, and the deviator stress, q, is assumed to be a unique surface in almost all the stress-strain theories developed at Cambridge University. However experimental observations provided by ROSCOE & THURAIRAJAH (1964) indicated that the state boundary surfaces obtained in the conventional triaxial apparatus were distinct and different atleast for the two common types of test conditions, namely, the undrained test and the fully drained test. In this paper the author has presented the data associated with the state paths followed by over 25 test specimens subjected to different testing conditions and varying applied stress paths.

STRESS PARAMETERS USED IN TRIAXIAL TESTS

The stress parameters used in the analysis of the triaxial test results are:

$$p = (\overline{\sigma}_1 + 2\overline{\sigma}_3)/3$$
 and $q = (\overline{\sigma}_1 - \overline{\sigma}_3)$, (since $\overline{\sigma}_2 = \overline{\sigma}_3$)

 $\overline{\sigma}_1,\,\overline{\sigma}_2$ and $\overline{\sigma}_3$ are the principal effective compressive stresses.

PREVIOUS WORK DONE ON STATE BOUNDARY SURFACES

Constant Voids Ratio Contours of RENDULIC (1936)

RENDULIC (1936) performed a series of stress controlled compression and

^{*} Assistant Professor of Geotechnical Engineering, Asian Institute of Technology, Bangkok, Thailand.

Discussion on this paper is open until 1 November 1974.

extension tests on remoulded saturated clay. The results were plotted in a $\overline{\sigma}_1$, $\sqrt{2}\overline{\sigma}_3$ space and Rendulic found that the stress paths followed by specimens sheared at constant voids ratio (undrained test) are in close agreement with the constant voids ratio contours derived from drained tests and isotropic consolidation tests. Further by assuming that these contours are reproducible in the $\overline{\sigma}_1 = \overline{\sigma}_3$ and $\overline{\sigma}_1 = \overline{\sigma}_2$ planes, Rendulic suggested that the constant voids ratio contours form surfaces of revolution about the space diagonal $\overline{\sigma}_1 = \overline{\sigma}_2 = \overline{\sigma}_3$ when plotted in the three-dimensional stress space.

Effective Stress, Water Content Relationships of Henkel (1960)

Henkel (1960) provided experimental results on Weald Clay and London Clay to illustrate the existence of a unique relationship between the effective stresses and the water content for each clay. The compression test results provided by Henkel are of three types with stress paths of slope, dq/dp, equal to -3/2, 3 and ∞ respectively. Similar stress paths are also imposed on the extension side. His results indicated that the constant water content contours obtained from drained tests are similar in form to the stress paths corresponding to the conventional undrained tests.

(p, q, e) Surface of ROSCOE et al (1958)

Roscoe et al (1958) put forward a basic concept of the existence of a unique surface in the three-dimensional space relating the voids ratio e, and the stress parameters q and p. Such a surface for normally and lightly overconsolidated clays is represented by ABCD in Fig. 1 which is based on triaxial test data from Imperial College on Weald Clay. The surface CDEF corresponding to overconsolidated specimens is based on less reliable evidence than that supporting ABCD. Also these authors suggested that it is not possible for the specimen to exist at a state (defined by p, q and e) outside the domain bounded by the two surfaces ABCD and CDEF unless tensile stresses are imposed. These two surfaces are subsequently called the state boundary surface. In describing the surface ABCD in Fig. 1 Roscoe et al (1958) only used the results of undrained tests.

Similarity of Undrained Stress Path and State Boundary Surface in Two-Dimensional Plot

Assuming that all the e = constant sections of the state boundary surface are geometrically similar ROSCOE & POOROOSHASB (1963) transformed this three-dimensional surface relating (p, q and e) into a two dimensional curve. The

STATE BOUNDARY SURFACE OF CLAY

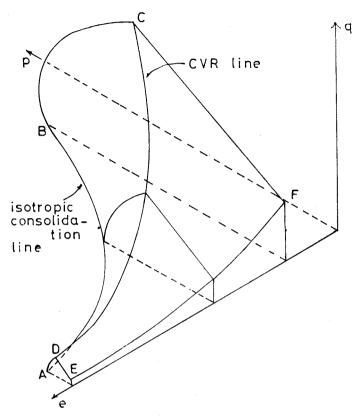


Fig. 1. (p, q, e) surface of ROSCOE et al (1958).

relevant parameters selected were q/p and $\frac{1}{p} \exp\left(\frac{e_a-e}{\lambda}\right)$ denoted by η and ξ respectively. The function $\exp\left(\frac{e_a-e}{\lambda}\right)$ is equivalent to the parameter p_e defined by Hvorslev (1937). In this relationship e_a corresponds to the voids ratio at unit pressure on the virgin consolidation line and p_e is the equilibrium pressure corresponding to the instantaneous voids ratio e on this line. Subsequent workers [e.g. Burland (1965)] have used the alternative parameters q/p_e and p/p_e for the two-dimensional representation of the state boundary surface. A unique curve can only be expected in this two dimensional plot for that part of the state path followed in any type of triaxial test that lies on the state boundary surface representing the state paths obtained from undrain-

Uniqueness of State Boundary Surface, ROSCOE & THURAIRAJAH (1964)

ed tests.

A detailed investigation of the state paths followed by specimens in the $[\eta, p \exp(e/\lambda)]$ space for tests carried out on Kaolin and other clays were

reported by Roscoe & Thurairajah (1964). For Kaolin in the triaxial apparatus these authors found that the drained and undrained test gave different curves in the $[\eta, p \exp(e/\lambda)]$ space (see Fig. 2). The values of $p \exp\left(\frac{e}{\lambda}\right)$

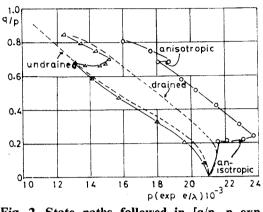


Fig. 2. State paths followed in [q/p, p] exp (e/λ) space of specimens sheared along undrained, fully drained and anisotropic consolidation paths (after ROSCOE & THURAIRAJAH, 1964).

corresponding to a given value of q/p was greater for the drained test path than for the undrained. The state paths followed during anisotropic consolidation (i.e. consolidation) under constant stress ratio $\eta = q/p$ seem to deviate from the drained and undrained surfaces. This deviation was smaller for anisotropic consolidation paths initiated from the drained test path at high stress ratios. These observations will be discussed further in the light of authors' test results.

SAMPLE PREPARATION AND TESTING PROCEDURE

Air dried kaolin (liquid limit = 74 %, plastic limit = 42%, plasticity index = 32% and specific gravity = 2.61%) was mixed with 160% distilled water and was prepared in the form of a slurry. This slurry was one-dimensionally consolidated under an axial stress of 22 lb/in² and subsequently isotropically consolidated to the relevant levels of isotropic stress in the triaxial apparatus. The sample former used in the one-dimensional consolidation and other details are given by BALASUBRAMANIAM (1969).

STATE PATHS FOLLOWED BY SPECIMENS PREPARED UNDER DIFFERENT ONE-DIMENSIONAL CONSOLIDATION STRESS

Fully Drained Test Results

Figure 3 illustrates the state path followed by three specimens AU, AB and AR which were prepared under initial one-dimensional consolidation stresses of 11 lb/in^2 , 22 lb/in^2 and 55 lb/in^2 respectively. The state paths in the $(q/p_e, p/p_e)$ space are found to be different. Also the specimen with the initial one-dimensional stress close to the isotropic stress of 60 lb/in^2 (i.e. specimen AR) has the highest value of p/p_e for any specific value q/p_e . It can be shown that specimens which have lower volumetric strains (at any particular stress ratio) will have higher values of p/p_e for any specific q/p_e .

STATE BOUNDARY SURFACE OF CLAY

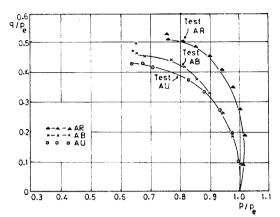


Fig. 3. State paths followed by specimens prepared from different initial one-dimensional stress and subsequently sheared from the same isotropic stress under fully drained conditions.

Undrained Test Results

The observations during undrained tests on two samples T_7 and T_4 will now be presented. Specimen T_7 was prepared from a slurry by initial one dimensional consolidation under a vertical stress of 3 lb/in², which corresponds to an applied deviator stress of 1 lb/in². This specimen was then isotropically consolidated under a pressure of 60 lb/in² and was then sheared in a conventional undrained triaxial compression test. Sample

 T_4 was subjected to a similar treatment but was given more initial one dimensional consolidation since it was subjected to a vertical stress of 22 lb/in² corresponding to an applied deviator stress of 6.6 lb/in².

the state of Figure 4 illustrates boundary surfaces for the two specimens T₇ and T₄. The undrained state path corresponding to specimen T₇ which had only experienced a previous shear stress of 1 lb/in² during one-dimensional consolidation was much higher than that for T_4 (previously subjected to a shear stress of 6.6 lb/in²). would, therefore, seem that if no allowance is made for stress or strain history, the undrained state boundary surface is not unique but does depend on the stress history of the sample.

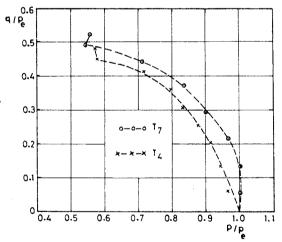


Fig. 4. State paths followed by specimens prepared from different initial one-dimensional stress and subsequently sheared from the same isotropic stress under undrained conditions.

STATE PATHS FOLLOWED BY SPECIMENS SHEARED WITH DIFFERENT LOAD INCREMENT SIZES

Fully drained compression tests at constant cell pressures with large increments can cause high pore pressures to develop and hence the specimen will be virtually subjected to an undrained stress path and a constant q stress path.

Experimental observations on four fully drained tests are presented in this section. The specimen Z was subjected to stress increments of 0.7 lb/in^2 at three hour intervals. Five stress increments were applied during a day and the equilibrium readings were then taken before applying the next increment in the following morning. In test R stress increments of 7 lb/in^2 were applied at two days intervals. Specimen X was subjected to stress increments of 14 lb/in^2 at two days intervals and in test Y a single stress increment of 26 lb/in^2 was used. The $(q/p_e, p/p_e)$ characteristics of these specimens are

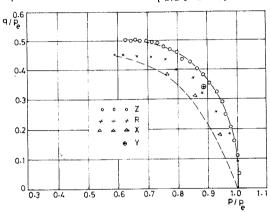


Fig. 5. State paths followed by specimens prepared under identical conditions and sheared from the same isotropic stress state with constant cell pressure but with varying load increment sizes.

indicated in Fig. 5. It is seen that the state paths followed by specimens with large increments (namely R, X and Y) always lie between the undrained state path and the path followed by the specimen Z which had a large number of small increments. It is suggested that if there had been no effects of the initial one-dimensional consolidation stress then there would have been a unique $(q/p_e, p/p_e)$ characteristic independent of the load increment size.

STATE PATHS FOLLOWED BY SPECIMENS SHEARED WITH DIFFERENT LOAD INCREMENT DURATIONS

Effects of time on the state boundary surface of fully drained tests are presented in this section. The uniqueness of the state path followed by specimens in a continuous stress controlled test is investigated by carrying out fully drained tests from 60 lb/in² cell pressure. Three specimens of Kaolin are tested. These specimens have load increment durations of 1/2 a day (specimen AB), 1 day (specimen W) and two days (specimen R). From ten to twelve increments of load are

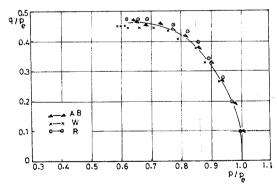


Fig. 6. State paths followed by specimens prepared under identical conditions and sheared from the same isotropic stress state with constant cell pressure and load increments but with varying load increment durations.

STATE BOUNDARY SURFACE OF CLAY

applied in each test and, therefore, the duration of the test varied from 5 to 6 days to about 20-24 days. Figure 6 illustrates the state paths followed by these specimens in the $(q/p_e, p/p_e)$ coordinates. From these observations it is clear that the state boundary surface corresponding to load increment durations of half a day or greater is approximately unique.

STATE PATHS FOLLOWED BY SPECIMENS SHEARED AT DIFFERNET ISOTROPIC STRESS

In this section the author has studied the state paths followed by specimens sheared from isotropic stress levels of 30, 60 and 90 lb/in² when subjected to three different types of imposed stress paths. The stress paths were those of (i) an undrained test, (ii) a constant-p test and (iii) a fully drained test. All specimens were initially prepared under a one-dimensional consolidation stress of 22 lb/in² and subsequently isotropically consolidated to the relevant isotropic stress.

First the state paths followed by specimens tested under undrained

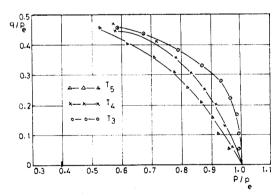


Fig. 7. State paths followed by specimens prepared under the same initial one-dimensional stress, but sheared from different isotropic stress states with constant cell pressure.

conditions will be presented. characteristics $(q/p_e, p/p_e)$ three specimens are shown in Fig. 7. In this figure specimen T₅ corresponds to the sample that was isotropically consolidated to 30 lb/in^2 . Specimen T₄ was isotropically consolidated to 60 lb/in² and the sample T₃ to 90 The state paths followed lb/in². by these three specimens are significantly different. The state paths are found to move outwards away from the origin as the level of isotropic stress is increased.

Now the state paths of three specimens sheared under constant p condition will be considered. Similar to the undrained tests these specimens were also isotropically consolidated to 30 lb/in² (specimen AJ) 60 lb/in² (specimen AQ) and 90 lb/in² (specimen AO). The state paths of these specimens are illustrated in Fig. 8. Unlike in the case of undrained tests, the state paths corresponding to the constant-p conditions are such that as the level of isotropic stress is increased the state paths move towards the origin. The outermost surface correspond to the lowest isotropic stress and the innermost

surface corresponds to the highest isotropic stress. It is interesting to note that the $(q/p_e, p/p_e)$ characteristic of the specimen with isotropic stress

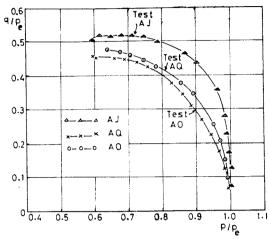


Fig. 8. State paths followed by specimens prepared under the same initial one-dimensional stress, but sheared from different isotropic stress states with constant-p stress paths.

of 30 lb/in² and one-dimensional consolidation stress of 22, lb/in² (i.e. specimen AJ) is approximately the same as that shown in Fig. 3 for the specimen AR which was again subjected to a similar high ratio of one-dimensional to isotropic stress (for specimen AR the initial one-dimensional stress was 55 lb/in² and the subsequent isotropic stress was 60 lb/in²). Thus the ratio of initial one-dimensional stress to the isotropic stress for specimen AR is 55/60=0.9; the corresponding ratio for the specimen AJ in Fig. 8 is 22/30 = 0.73

Finally the results of three test specimens sheared under fully drained conditions (i.e. with constant cell pressure and dq/dp = 3) from isotropic stresses of 30, 60 and 90 lb/in² will be presented. The specimen AF was isotropically consolidated to 30 lb/in² the specimen Z to 60 lb/in² and the specimen AD to 90 lb/in². The state paths followed by these samples are shown in Fig. 9.

These results too confirmed the finding from the constant-p tests in that the specimen subjected to the lowest isotropic stress has a higher state path than the other Also, the diftwo specimens. ference between the specimen AF which was consolidated to 30 lb/in², from that of specimen Z consolidated to 60 lb/in² is much more pronounced than the corresponding difference in specimens Z and AD. The reason for this observation will be clear once the results presented in Fig. 10 are studied in detail. This figure shows the experimentally

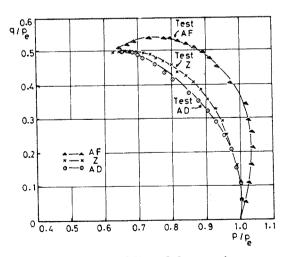


Fig. 9. State paths followed by specimens prepared under the same initial one-dimensional stress, but sheared from different isotropic stress states under fully drained conditions.

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observed $(q/p_e, p/p_e)$ curves for all the three types of tests (i.e. undrained, constant-p and fully drained test) at each of the three isotropic stresses of 30 lb/in², 60 lb/in² and 90 lb/in². All the samples had been previously one-dimensionally consolidated under a vertical stress of 22 lb/in². The maximum deviation between the three types of stress paths is observed in the case of tests on the samples consolidated at a 30 lb/in² isotropic stress; the extent of this type of deviation diminishes as the isotropic stress increases from 30 to 90 lb/in². The behaviour at 90 lb/in² is approximately unique for all three types of test.

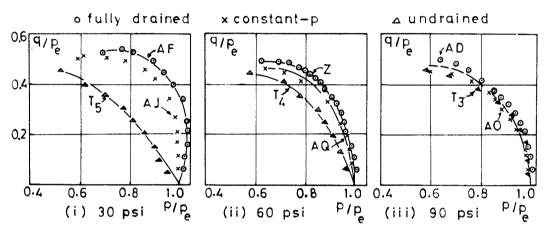


Fig. 10. State paths followed by specimens prepared under the same initial one-dimensional stress but sheared under undrained, constant-p and fully drained conditions from each of the three isotropic stresses of 30, 60 and 90 lb/in².

These observations may be compared with those of ROSCOE & THURAIRA-They too noted a difference in the state boundary surface folјан (1964). lowed by the specimens in the drained and undrained tests on Kaolin when plotted in the alternate plot $[q/p, p \exp(e/\lambda)]$ which is similar to the (q/p_a) p/p_e) plot. The one-dimensional stress used in the preliminary preparation of samples by Thurairajah (1961) was of the order of 33 lb/in² which is much higher than the value of 22 lb/in² used by the present author. Hence, the difference observed by Roscoe & Thurairajah (1964) in the $[q/p, p \exp(e/\lambda)]$ plot of the drained and undrained state paths in triaxial specimens of Kaolin can be attributed to the predominant effect of the initial one-dimensional consolidation stress. Measurements of anisotropic strains during isotropic consolidation in Kaolin carried out by BALASUBRAMANIAM (1969) suggests that the effects of the initial one-dimensional stress could only be errased by isotropically consolidating the sample to a value about three times the initial one-dimensional stress used in sample preparation.

From the above results it is apparent that provided the effect of initial one-dimensional consolidation stress has not been there, the state boundary surface is unique at least for the undrained, constant-p and fully drained conditions. This unique state boundary surface is best represented by the results corresponding to the 90 lb/in² isotropic stress in Fig. 10. This approximate unique state boundary surface is used as a standard state boundary

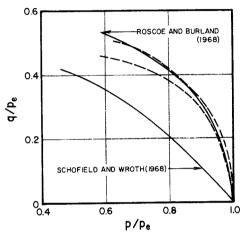


Fig. 11. Approximate unique state boundary surface for specimens sheared under undrained, constant-p and fully drained conditions from 90 lb/in² isotropic stress.

surface of normally consolidated specimens of Kaolin for comparison with the state paths for all other tests which are carried out to investigate the effects of stress paths. The scatter in the results is allowed for by representing the surface with a double boundary as illustrated in Fig. 11.

The state boundary surfaces as predicted by the stress-strain theories of Roscoe & Burland (1968) and Schofield & Worth (1968) are also shown in Fig. 11. The theoretical model of Roscoe & Burland is found to predict the state boundary surface remarkably well.

STATE PATHS FOLLOWED BY SPECIMENS SHEARED UNDER WIDE VARIETY OF STRESS PATHS

The state paths considered so far always corresponded to the conditions that during an applied stress path the deviator stress q and the stress ratio q/p have always been increasing from the maximum preconsolidation pressure. For these cases, a unique state boundary surface was found as that presented in Fig. 11. Now it will be interesting to consider other types of stress paths such as:

- (i) anisotropic consolidation, where the stress ratio q/p is maintained constant;
- (ii) constant-q, where the deviator stress is maintained constant;
- (iii) decreasing q, with a consequent decrease in stress ratio q/p;
- (iv) those corresponding to extension condition, where the principal axes of stresses are rotated from those corresponding to the compression condition; and
- (v) isotropic swelling (corresponding to overconsolidated samples) followed by stress paths with increasing q and q/p.

STATE BOUNDARY SURFACE OF CLAY

In all these cases, the stress paths followed by the specimens will be presented together with the state paths for clarity of the presentation of results.

State Paths Followed during Anisotropic Consolidation

The stress-strain theory of Roscoe & Poorooshasb (1963) assumes that the state paths during anisotropic consolidation lie on the state boundary surface. The subsequent theories due to Roscoe et al (1963) and Roscoe & Burland (1968) also imply that the state paths during anisotropic consolidation lie on the state boundary surface. However, the experimental observations provided by Roscoe & Thurairajah (1964) indicated that the state path corresponding to anisotropic consolidation does not lie on the same state boundary surface as that corresponding to the undrained or drained test. Therefore, the present investigation carried out by the author would help to resolve whether the state path corresponding to anisotropic consolidation does or does not lie on the unique state boundary surface obtained in Fig. 11 from undrained, constant-p and fully drained tests.

Several anisotropic consolidation tests were carried out but experimental observations will only be provided on two samples designated BC and BL which are representative of all the tests. Sample BC was sheared from 90 lb/in^2 isotropic stress along an undrained stress path until the stresses q and p were 32 lb/in^2 and 77 lb/in^2 respectively. Thereafter, the specimen was sheared along an anisotropic consolidation path with the stress ratio q/p being maintained at a constant value of 0.42. The stress path and the state path followed by this sample is shown in Fig. 12. The state path corresponding to anisotropic consolidation is found to lie on the state boundary surface derived for normally consolidated clays (see Fig. 11). The stress condition of

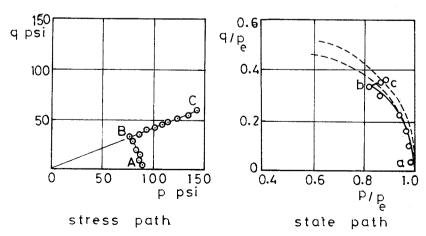


Fig. 12. Stress path and state path followed by specimen BC during anisotropic consolidation.

specimen BC at the end of anisotropic consolidation was q = 57, $1b/in^2$ and $p = 145 lb/in^2$.

Figure 13 illustrates the stress path and the state path followed by specimen BL. This sample was subjected initially to a convex stress path from its isotropic consolidation pressure of 90 lb/in² until the stresses q and p were 65 lb/in² and 110 lb/in² respectively. From this stress level it was subjected to an isotropic consolidation with q/p = 0.58 till q = 105 lb/in² and p = 180 lb/in². The state path followed by this sample was also found to lie on the state boundary surface for normally consolidated clay specimens. The results presented here would, therefore, indicate that irrespective of the shape of the initial stress path followed, all specimens have their state path on the state boundary surface during anisotropic consolidation. These results are in contradiction to those of Roscoe & Thurairajah (1964) and are in agreement with the stress strain theory of Roscoe & Poorooshasb (1963), Roscoe et al (1963) and Roscoe & Burland (1968).

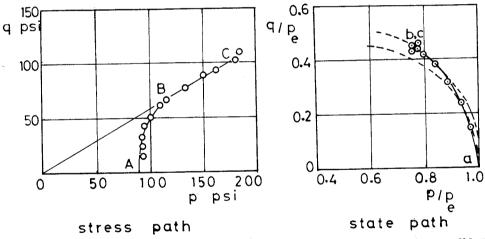


Fig. 13. Stress path and state path followed by specimen BL during anisotropic consolidation.

State Paths Followed by Specimens Sheared along Constant-q Stress Paths

Constant-q stress paths have important applications in the field in the sense that when a large increment of load is applied to a layer of clay, initially the layer would be subjected to undrained stress conditions and there would be an increase in the pore pressure. The subsequent dissipation of pore pressure will be under constant deviator stress and, therefore, will be the same as that of a constant-q stress path. Several constant-q tests were performed, but only the observation on one sample T_{13} will be presented here. The observation provided for sample T_{13} is representative of all the samples tested under constant-q condition.

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Specimen T_{13} was initially subjected to a constant-p stress path from an isotropic stress of 90 lb/in² until the stresses were q=30 lb/in² and p=90 lb/in². This constant-p stress path was followed by a constant-q stress path until the mean normal stress reached a value of 117 lb/in². Finally, this specimen was sheared to failure by an applied stress path of slope 3 (i.e. fully drained condition with constant cell pressure). The stress and state path followed by this specimen is shown in Fig. 14 and the state path is found to lie on the state boundary surface derived from undrained, constant-p and fully drained tests.

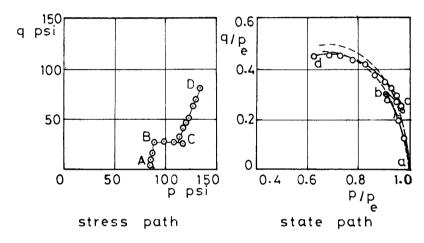


Fig. 14. Stress path and state path followed by specimen T₁₃ during constant-q path.

State Path Followed by Specimens Sheared under Decreasing Deviator Stress Condition

Experimental observation will only be presented on one sample, CO, where the behaviour exhibited is typical of several other samples tested under similar conditions. Sample CO was sheared under constant-p condition from 90 lb/in² isotropic stress up to a stress level of q = 17 lb/in² and p = 90 lb/in². Thereafter, the specimen was subjected to a stress path with decreasing deviator stress and increasing mean normal stress until the stresses are q = 2 lb/in² and p = 145 lb/in². Finally the specimen was sheared along a fully drained stress path with slope dq/dp = 3. The stress path followed by this specimen and the state path are indicated in Fig. 15. It is noted that the state path corresponding to the stress path followed by specimen CO fully lies on the state boundary surface derived from the undrained, constant-p and fully drained tests.

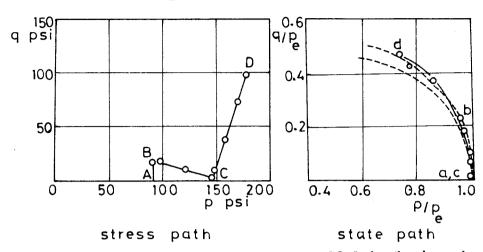


Fig. 15. Stress path and state path followed by specimen CO during the phase when the deviator stress q was decreased.

State Paths Followed by Specimens of Kaolin in Extension Tests

The experimental observations provided so far have been from compression tests performed on normally consolidated specimens of Kaolin. During compression test the major principal stress is along the axis of the sample. It will now be interesting to observe the state paths followed by specimens in extension tests. During extension the lateral stresses are the major principal stress and, therefore, any effect due to initial anisotropy in the sample developed as a result of the initial one-dimensional consolidation will be shown pronouncedly in specimens of Kaolin sheared under extension.

Experimental observation will only be presented on one sample, BG, which was sheared from an isotropic stress of 60 lb/in^2 along an applied stress path of slope 3. In presenting the data for extension test, the deviator stress q would be considered as $\sigma_3 - \sigma_1$ instead of being $\sigma_1 - \sigma_3$. This procedure is only adopted to avoid a negative sign in the values of q during extension tests. The stress path and the state path followed by this sample are shown in Fig. 16. The state path of this specimen is also found to lie on the state boundary surface derived from undrained, constant-p and fully drained tests. The result presented for specimen BG is representative of the behaviour of several other samples tested under similar conditions.

From the experimental observations of Parry (1956, 1960) Roscoe & Poorooshasb (1963) suggested that the state boundary surface of specimens sheared under extension conditions with $\overline{\sigma}_1$ = constant and $\overline{\sigma}_3$ decreasing could be different from those of the specimens sheared under conditions of p increasing. The author's results of extension tests, though not carried out

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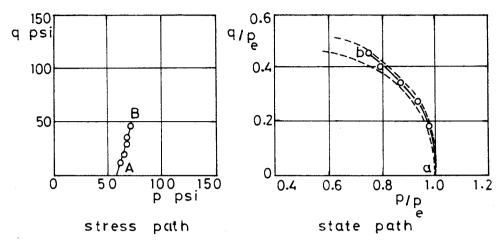


Fig. 16. Stress and state path followed by specimen BG during extension test.

under the above conditions, indicate that the state boundary surface is unique for specimens sheared under both compression and extension conditions.

LELIEVRE & WANG (1971) presented the results of compression extension tests indicating the effect of strain-hardening with respect to distortional strains. It would be interesting for future research workers to study such secondary effects on the state boundary surface.

State Paths Followed by Lightly Overconsolidated Samples

Experimental observations will now be presented on two lightly overconsolidated samples, BD and BJ, which have been isotropically consolidated to 90 lb/in² and then swollen back to 56 lb/in² under isotropic conditions. Specimen BD was subjected to a fully drained stress path of slope 3. The stress and state path followed by this sample are shown in Fig. 17. It is noted that

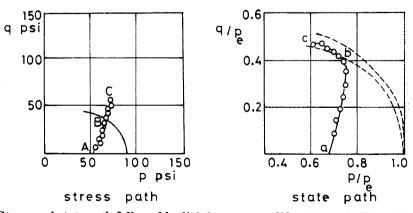


Fig. 17. Stress and state path followed by lightly overconsolidated sample BD under constant cell pressure condition.

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the state path rises from the p/p_e axis and lies inside the state boundary surface for all values of stresses corresponding to points which lie inside the undrained stress path passing through the point q=0 and p=90 lb/in². For states of stress lieing outside this undrained stress path the state path is found to lie on the state boundary surface. This observation is further confirmed by the behaviour of specimen BJ which has been subjected to an applied stress path of slope 1.5. These results are presented in Fig. 18.

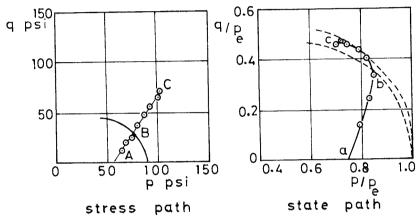


Fig. 18. Stress path and state path followed by specimen BJ sheared from a lightly overconsolidated state along an applied stress path of slope 1.5.

The above results would indicate that, for the states of stress which lie inside the undrained stress path through the maximum preconsolidation pressure, the state paths lie inside the state boundary surface. For the states of stress which lie outside or on the undrained stress path the state paths lie on the state boundary surface.

CONCLUSIONS

Fully drained compression tests (with constant cell pressure) on specimens prepared under different initial one-dimensional consolidation stress and subsequently consolidated to the same isotropic stress showed that the state paths followed in $(q/p_e, \, p/p_e)$ space are dependent on the magnitude of the initial one-dimensional consolidation stress. A set of undrained tests performed in a similar manner revealed the same finding.

Factors such as the load increment size and the load increment duration do not seem to have appreciable effect on the state boundary surface provided there is no effect of initial one-dimensional consolidation stress.

Three types of test (undrained test, constant-p test, and fully drained test) carried out on specimens prepared under the same initial one-dimensional

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stress condition and subsequently sheared from three different levels of isotropic stress indicated that the state paths followed in the $(q/p_e, p/p_e)$ space are dependent on the magnitude of the isotropic consolidation stress. At any one particular isotropic stress the largest deviation is observed for tests conducted at the lowest consolidation pressure and those conducted at the highest consolidation pressures were virtually identical.

For the following types of applied stress paths imposed on normally consolidated specimens of Kaolin, the state paths were found to lie on the unique state boundary surface derived from undrained, constant-p and fully drained tests conducted on specimens isotropically consolidated to 90 lb/in².

- (i) anisotropic consolidation paths;
- (ii) constant-q path;
- (iii) stress path with decreasing q; and
- (iv) extension paths.

A series of tests performed on lightly overconsolidated samples indicated that the state path lies inside the state boundary surface for all states of the stress that lie inside the undrained stress path through the maximum preconsolidation pressure. For the states of stress which lie outside the undrained stress path the state paths lie on the state boundary surface.

ACKNOWLEDGEMENTS

The work presented in this paper was carried out at the Engineering Laboratories, University of Cambridge when the author was a research student. The author wishes to thank his supervisor Dr. R.G. James and the late Prof. K.H. Roscoe for their invaluable guidance and unstinted help. The manuscript of this paper was prepared at the Faculty of Engineering, University of Sri Lanka, Peradeniya, Sri Lanka. Thanks are due to Prof. A. Thurairajah for his continuous support and encouragement. Mr. K. Kumarasubramaniam of the Soil Mechanics Laboratory has given considerable assistance in the preparation of this paper.

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DEFORMATIONS OF STATISTICALLY HETEROGENEOUS EARTH STRUCTURES

LELAND M. KRAFT, JR.* and JIMMY YEW-HANG YEUNG+

SYNOPSIS

An analytical investigation of the statistical uncertainty of earth structure deformation is reported. Soil slopes are modeled as isotropic linearly elastic materials, but the elastic properties vary spatially in a statistically heterogeneous manner. This investigation illustrates the use of the finite element technique and statistical concepts to estimate the uncertainty of deformations by simulating soil heterogeneity with a random distribution of moduli properties for the elements. Three excavated earth slope angles and three embankment slope angles were analyzed with three coefficients of variation of the elastic modulus; two values of Poisson's ratio were used for the majority of the cases studied. The results of the analyses provide a quantitative measure of the uncertainty in the deformations, as influenced by spatial variations within earth structures. The advantages to be gained by analyzing the slopes in this statistical manner are suggested with the results of this study.

INTRODUCTION

The physical soil properties from two adjacent boreholes are seldom if ever identical. It is recognized that soil properties are heterogeneous even in deposits of soil having the same general description of soil type throughout. Because of the heterogeneous nature of soils, and other factors, the soil engineer is faced with a difficult task of selecting representative soil properties from a limited amount of data for use in analyses. To account, at least in part, for the uncertainty induced in the results of the analyses by the uncertainty in the selected representative properties, and by the uncertainty due to the random spatial variation, safety factors are used. These safety factors are selected for use in design on the basis of engineering judgement and experience, which are deemed on an intuitive basis to result in an acceptably small likelihood of failure. The risks associated with a given sefety factor are not and cannot be evaluated quantitatively when safety factors are selected in In view of the many associated uncertainties a more rational approach to design is a quantitative evaluation of the uncertainties in terms of probability and statistics followed with a quantitative evaluation of the influence of these uncertainties on the risk.

^{*} Senior Engineer, McClelland Engineers, Inc., Houston, Texas, U.S.A.

⁺ Graduate Research Assistant, Case Western Reserve University, Cleveland, Ohio, U.S.A. *Discussion* on this paper is open until 1 November 1974.

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Since the term "safety factor" can take on many meanings, a universally accepted definition of the safety factor is required for interpreting the results from particular applications. As used here, "safety factor" implies only a concept and no attempt is made to relate the findings to any definition of safety factor. In fact, it is the lack of a one-to-one correspondence between safety factor and risk that justifies most the use of probability and statistics for design.

A great amount of effort has been expended in recent years in applying statistics and probability to analyses and designs in soil engineering problems (e.g., Lumb, 1966, 1967, 1968, 1970; Langejan, 1965; Wu & Kraft, 1967, 1970; FOLAYAN et al, 1970). Assumptions in these studies were generally required concerning the magnitude of the soil property which governs the The assumption most commonly made was either performance of the soil. that the mean value or that a lower extreme of a material property governs the performance. If a slope stability or bearing capacity problem is being analyzed it may be assumed, for example, that the mean strength along the The results of a comparison potential failure surface governs the stability. between measured and theoretical bearing capacity suggest that the arithmetic mean strength may be an unconservative lumped parameter (Lumb, 1968). Hence, the validity of such assumptions are open to question. A study of the deformations of statistically heterogeneous soil masses would be useful, as a first step, in examining the validity of the assumptions now made. It would also provide insight for the adoption of more appropriate assumptions for the selection of lumped parameters for use with conventional mathematical design predictive models.

This paper presents the findings obtained from a simulation study of statistical slope deformations of statistically heterogeneous, excavated earth slopes and earth embankments. The parameters studied in this work include the angle of the slope, cross-sectional shape, modulus of elasticity, Poisson's ratio, coefficient of variation in the modulus of elasticity, and slope height. Three slope angles, two Poisson's ratios, and three coefficients of variation of the elastic modulus are used for the majority of cases studied.

In this study, the mean soil stiffness is assumed to be known. However, since in practice mean soil properties are based on a limited number of samples, the mean is not known with certainty. Further uncertainty in the mean exists due to sampling and testing disturbance and differences between stress conditions in the field and in the laboratory. These additional uncertainties can be evaluated quantitatively through a simple extension of this work and will be the object of future studies. The results of such extensions should

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provide a quantitative measure to determine the relative significance that each of the mentioned uncertainties has on the total risk.

CONCEPTUAL MODEL

Figure 1 illustrates a random variation of the soil modulus, E, within a slope. The soil modulus between each element or group of elements can be different. The spatial uncertainty of the soil properties results in an uncertainty of predicted slope deformation, which can be estimated by making use of statistical concepts together with the finite element method of analysis. This approach will assist in evaluating the risk of failure associated with a design, and will provide for a more realistic approach to design using optimization methods. It will also assist in selecting design soil parameters from data obtained from a limited number of samples from a limited number of boreholes. Similar work was done by Levey & Barenberg (1971) using a discrete element method applied to a layered system, analysis to evaluate the statistical nature of stresses, strains and deflections in pavements composed of materials with variable physical characteristics.

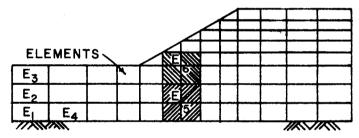


Fig. 1. Spatial variation of soil stiffness.

Statistical Input Parameters

A typical stress-strain curve for a soil is shown in Fig. 2. The slope of the line OA (which is the modulus of elasticity, E) can be used as an approximation of the stress-strain curve. This approximation is valid for some cohesive soils provided a safety factor against shear is on the order of 1.5 to 2.0. A linear approximation is used in this study; however, the concept can be extened to soils with a nonlinear stress-strain response.

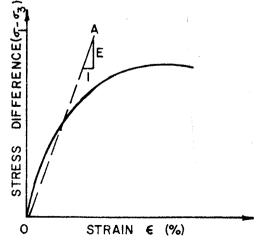


Fig. 2. Typical stress-strain response of a cohesive soil.

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Although statistical data on the modulus of soils is lacking, an abundance of data on soil strength are available. Studies have shown that, for a given cohesive soil, the elastic modulus may be approximately proportional to the shear strength (e.g. D'APPOLONIA et al, 1971). With a given relation between E and soil strength, the statistical behavior of the elastic modulus can be estimated from the statistical behavior of the soil strength. It has been demonstrated (Lumb, 1966; Wu & Kraft, 1970) that a lognormal probability distribution for soil strength is a reasonably good assumption. If the elastic modulus is proportional to the strength, the elastic modulus also follows a lognormal probability distribution. The results of this study are based on a lognormal probability distribution for the elastic modulus.

The only input random variable considered was the elastic modulus of the soil. If the elastic modulus is proportional to the undrained strength, the coefficient of variation of the strength. The coefficient of variation of a variable is the ratio of the standard deviation to the mean and is a measure of the relative dispersion, or uncertainty, in the variable. A coefficient of variation of zero implies no randomness, or in other words, complete certainty about the value of the variable. Studies on clay have shown that 0.3 is a typical value for the coefficient of variation of the undrained strength, and 0.6 is representative of an upper value (Lumb, 1966). The coefficient of variation of E selected for study ranged from zero (the deterministic case) to 0.6.

Random values of the elastic modulus were generated by a random number generator incorporated into the finite element program assuming a lognormal distribution of E with a known mean and variance. Each random E value generated was assigned to a different element, starting from the first element to the last element, or from the first group of elements to the last group of elements. After each element or group of elements had been assigned an E value, the finite element portion of the computer program was used to calculate displacements. This process was repeated 100 times for cases using the coefficient of variation of E, $V_{\rm e}$, equal to 0.3, and 200 times when $V_{\rm e}$ was equal to 0.6

Finite Element Mesh

The lateral extent of the slope region used in this work for the excavated slope problem was determined using the criterion suggested by Dunlop & Duncan (1969). The slope and embankment geometries studied are shown in Fig. 3. It was assumed that the embankment rested on bedrock. The angles of the slope for the excavation varied from about 18 to 90 degrees, and for the embankment from about 18 to 45 degrees. To estimate the precision

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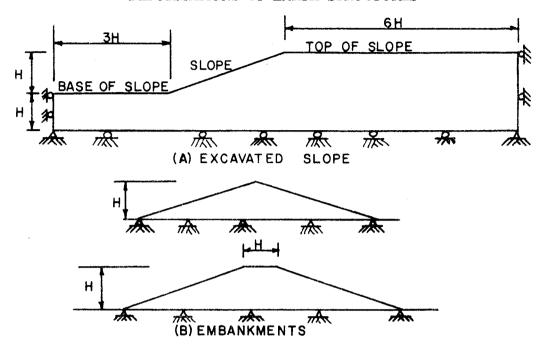


Fig. 3. Geometry of earth structures studied.

of the results provided by the finite element meshes used in this work, a total of three meshes were selected for homogeneous soil conditions; these were a coarse mesh (42 nodes), a medium mesh (121 nodes) and a fine mesh (408 nodes). The difference between the displacement results obtained for the medium mesh (one used in this work) and an extrapolated exact solution for the one-on-three slope ranged from approximately 1 to 10%, with discrepancy for the majority of nodal points being less than 5%. The larger difference occurred mainly at nodal points where the displacements were relatively small as compared with the largest displacement on the slope boundary. Further refinements in the accuracy could be made at the expense of computer time by utilizing a finer mesh. However, the error introduced by using a medium mesh was smaller than the error introduced by the uncertainty of soil properties due to sampling and testing errors. Thus, use of the medium mesh was deemed justified. Also, because of the comparative nature of the analyses as described later, numerical errors of a few per cent would tend to be cancelled.

Simulation of the excavation process was made by a one step unloading as described by Dunlop & Duncan (1970). The initial earth pressure coefficient K_o^* (in terms of total stress), ranged between 0.8 and 2.4. This range in K_o^* represents initial stress conditions typical of normally consolidated to moderately overconsolidated clays.

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PRESENTATION OF RESULTS

Only the surface deformations are evaluated, since the surface deformations fully reflect the deformations of points within the slope. An analysis of the results demonstrated that data could be represented by a normalized dimensionless parameter, $\frac{\delta \, \bar{E}}{\gamma \, H^2}$ in which γ is the soil unit weight and δ represents either the vertical displacement, δ_v , or horizontal displacement, δ_h . To obtain an overall picture of the influence of soil heterogeneity on the deformations along the surface of each slope, the following graphs were constructed for the horizontal and vertical displacements:

- (1) The normalized displacements for the equivalent homogeneous case (zero coefficient of variation, $V_{\rm e}$).
- (2) The ratio of the mean displacement for a statistically heterogeneous case to the displacement for the equivalent homogeneous case. This ratio, hereafter referred to as the "displacement ratio", provides a

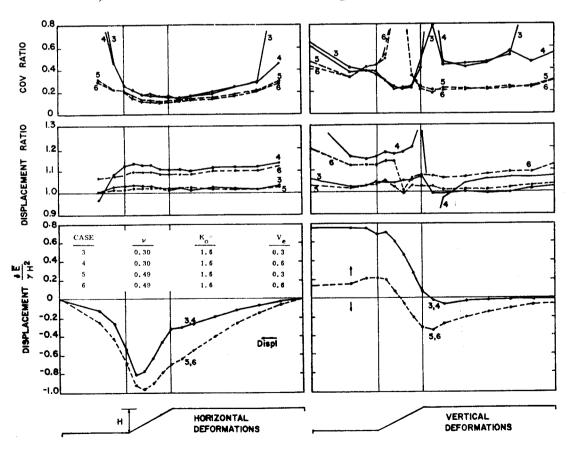


Fig. 4. Statistical deformation behavior of an elastic 1:2 slope.

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- measure of the relative influence of $V_{\rm e}$ on the displacement as compared to the homogeneous case.
- (3) The ratio of the absolute value of the coefficient of variation of the displacement to the coefficient of variation of the elastic modulus V_e . This ratio will hereafter be referred to as the "coefficient of variation" ratio (COV). The above variables were plotted against the surface of the slope as illustrated in Figs. 4 and 5 which are typical results of an excavation and embankment condition respectively. The normalized displacement in these figures is based on the equivalent homogeneous case (i.e. $V_e = 0$).

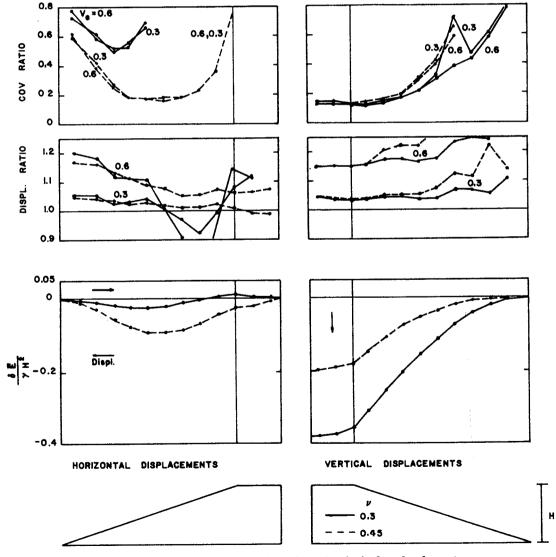


Fig. 5. Statistical deformation of an elastic 1:3 embankments.

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Since only a limited number of deformation samples were obtained in calculating the mean, $\bar{\delta}$, and the coefficient of variation of the deformation, V_{δ} , both $\bar{\delta}$ and V_{δ} are statistical estimates. In order to determine the statistical accuracy in estimating the mean displacement, the confidence interval in the majority of the cases was less than $\pm 5\%$ of the calculated mean horizontal and mean vertical displacement with a 0.95 confidence level for the number of sample deformations obtained. The confidence bands were found higher than 5% of the mean displacements only in regions on the slope having relatively very small deformations. To increase the statistical confidence above the 95% level would require an additional 200 or more samples, since the uncertainty of the unknown mean displacement decreases approximately as the reciprocal of the square root of the number of samples.

Because $\bar{\delta}$ and V_{δ} are only statistical estimates, considerable care must be made when drawing conclusions from data similar to that shown in Figs. 4 and 5. For example, the displacement ratio may vary radically between values less than 1.0 to values much greater than 1.0 in some regions of the slope. This erratic behavior, which is in all likelihood a result of an insufficient number of sample deformations and not an indication of the actual behavior, occurs in regions where the component of displacement is near zero. In these regions the coefficient of variation is large, and thus, for a given number of statistical samples the confidence interval is large. In fact, at some point on the slope, the mean vertical displacement is zero, which results in an infinite coefficient of variation of the vertical displacement. The significance of a coefficient of variation decreases as its mean approaches zero.

Displacement Ratio

The effect of the statistical heterogeneity of the soil stiffness on the slope deformations was studied by plotting the displacement ratio against the coefficient of variation ratio. The influence of $V_{\rm e}$ on the displacement ratio at the toe of the excavations and at the crest of the embankments is shown in Fig. 6, where it is observed that the displacement ratio increases approximately with the square of $V_{\rm e}$. For values of $V_{\rm e}$ less than approximately 0.2, the expected displacement of a statistically heterogeneous excavation can be predicted using the average value of soil stiffness in an analysis of a homogeneous soil profile. As $V_{\rm e}$ increases beyond about 0.2, the mean displacement of the statistically heterogeneous soil profile is larger than that calculated using an average stiffness and a homogeneous soil profile. Larger mean deformations associated with larger coefficients of variation indicated that the softer or weaker elements had a more influential effect on the performance of

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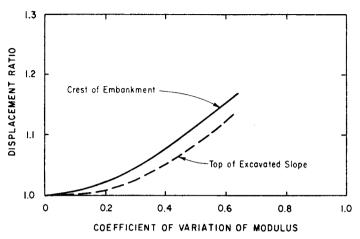


Fig. 6. Variation of the displacement ratio.

the earth structure than the stiffer elements, which is in agreement with experimental results on bearing capacities obtained by LUMB (1968).

Although the increase in mean displacement ratio may appear to be small (in the order of 10 to 20%), this increase can result in a significant increase in risk, especially for earth structures where low safety factors are common. The risk is affected by the displacement coefficient of variation as well as the mean displacement. If, for example, the displacement coefficient of variation is 0.12 for $V_e = 0.6$ ($0.12 = \text{COV} \times V_e$: see Figs. 4 and 5), the probability that the observed displacement will exceed 1.5 times that calculated using the equivalent homogeneous case is in the order of 5%. The additional uncertainty in the calculated mean stiffness could further increase this probability (or risk) significantly, as will be shown in subsequent papers extending this work. Although the assumption of elasticity (made in the analysis presented) may be challenged for low safety factors, the results are thought to provide a reasonable quantitative measure of the influence of soil heterogeneity on the performance of earth structures.

The results shown in Fig. 6 are based on conditions for the slope angles studied at the toe of the excavation and at the crest of the embankment. Similar behavior was found for other points on the slope, although quantitatively the results may differ.

Coefficient of Variation Ratio

If the risk of failure in terms of deformation is to be calculated, at least two statistical parameters of the deformation are required; namely, the mean and standard deviation (or alternatively the coefficient of variation) of the displacement. The influence of statistical soil heterogeneity on the mean

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displacement has been presented above. Rather than study the standard deviation, the authors examined the coefficient of variation. If the mean is known, the standard deviation can be calculated from the coefficient of variation. One advantage of working with the coefficient of variation is that it is a dimensionless quantity expressing the relative degree of dispersion of the random variable. The only disadvantage of using the coefficient of variation is that it becomes meaningless as the mean approaches zero. Fortunately, regions of the slope surface having relatively small or near zero deformations are not of primary interest.

It has been found that the coefficient of variation of the displacement, V_{δ} , is proportional to $V_{\rm e}$. Hence, V_{δ} is normalized by dividing the absolute value of V_{δ} by $V_{\rm e}$; the resulting ratio in this study is called the "coefficient of variation" ratio (COV). Values of COV less than 1.0 indicate a damping effect on the uncertainty of deformation; whereas, values of COV greater than 1.0 indicate an accentuating effect. In regions of the slope where the mean displacements were relatively large, the values of COV were less than 1.0. Only in regions of the slope where the mean displacements were relatively small (in absolute value) were the values of COV larger than 1.0. This behavior is obvious from Figs. 4 and 5 as well as from the data shown in Fig. 7, which shows the influence of $V_{\rm e}$ on the COV for two points on an excavated slope.

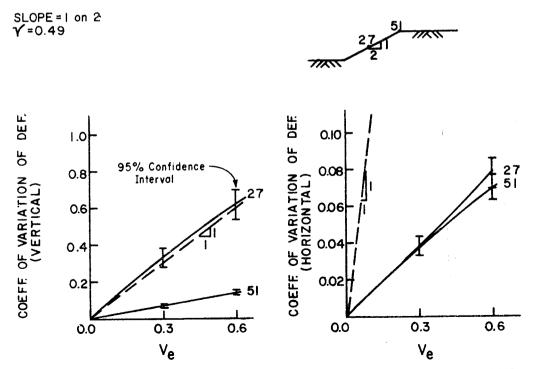


Fig. 7. Influence of the elastic coefficient of variation on the coefficient of variation of deformations for an excavated slope.

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Influence of Non-Uniform Mesh Size

The element size of the finite element mesh used in this study increased with distance away from the slope. To determine the influence of this non-uniform mesh size on the statistical behavior of the deformations a more uniform size mesh was used for a few selected cases. The results showed that the mesh size in regions distant from the slope did not affect the mean displacement of the slope significantly. Hence, the conclusions based on the non-uniform mesh used here were considered to be representative of results that would as obtained with a uniform mesh size.

Influence of Poisson's Ratio

The mean deformation is influenced by the magnitude of Poisson's ratio, as would be expected. However, the displacement ratio is not significantly affected by the value of Poisson's ratio. For the vast majority of cases, the uncertainty in the deformation as reflected by the coefficient of variation of the deformation was affected by the magnitude of Poisson's ratio. The deformation coefficient of variation increased with a decrease in Poisson's ratio (see Figs. 4 and 5).

Influence of K_o*

Although the value of K_0^* , which reflects the initial stress conditions, affects the magnitude of deflection, the displacement ratio and COV ratio were not noticeably affected by changes in K_0^* .

COMPARISON WITH MEASURED RESULTS

Throughout this study, the primary concern was to determine the influence of the magnitude of material uncertainty due to spatial variations on both the mean deformation and the uncertainty in the deformations. It is the comparative aspects of the statistical numbers that are of interest and not the individual magnitudes of the deformation.

To lend support to the analytical procedure of this study, the literature was searched to find measured slope deformation data. Although deformation data are available, the amount on identical or similar slope geometries is scarce. Hence, there are insufficient field data at this time to calculate measured means and standard deviations of slope deformations. Since more data are available on footing settlements, an analytical statistical evaluation of footing settlement for a particular case was carried out.

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The measured results of footing settlements are based on data given by D'APPOLONIA et al (1968) for footings on sand. The coefficient of variation of the standard penetration data was approximately 0.3. If the settlements are based on an equivalent elastic modulus which is proportional to the standard penetration number (SCHMERTMANN, 1970), the coefficient of variation of the elastic modulus also has a value of 0.3. The coefficient of variation of the measured settlement varied between 0.2 to 0.4, depending on the size and depth of the footing and the design pressure. Since the actual loads undoubtedly varied, some of the measured variation in the settlement is attributed to variations in the loads.

Using simulation with finite elements, the coefficient of variation of settlement, neglecting uncertainties in the loads, was found to be 0.05. This value is less than the measured value, as would be expected. If the soil were homogeneous, a coefficient of variation of load would result in an approximately equal coefficient of variation of settlement for a linear, or approximately linear, settlement model. Based upon reported measured variations in loads (e.g. Wu & Kraft, 1967), the difference between the predicted settlement variation and measured variation could be attributed to uncertainty in the loads. In the case of earth structures subjected only to gravity, however, the loads can be taken as deterministic.

Additional field data are needed before a conclusive evaluation can be made of the coupling of finite elements and simulation to predict uncertainties in designs due to spatial variations in soil properties. Intuitively, the method appears valid if properly used and the comparison given here indicates that there is at least general agreement in respect of the order of magnitudes.

SUMMARY

Statistical concepts and the finite element method were applied to slopes composed of linear isotropic but statistically heterogeneous elastic materials. Typical results were presented and discussed. It was demonstrated that this statistical, finite element method of analysis can provide an indication of the statistical uncertainty in the deformation of a slope with linear elastic material. With a knowledge of this uncertainty in the deformation, the design of a slope may be approached in a more efficient and rational fashion to achieve an optimum design, and it may lead to a quantitative evaluation of the influence of localized weak zones in the selection of safety factors.

The results of this study provide a quantitative measure of the influence that the soil heterogeneity, as measured by the coefficient of variation of the elastic modulus, has on the slope deformation. The mean deformation

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increases approximately with the square of the coefficient of variation of the elastic modulus, and for $V_{\rm e}=0.6$, the mean displacement may be 20% greater than for $V_{\rm e}=0$. The uncertainty of deformation, as reflected by the coefficient of variation of the deformation, is approximately proportional to the coefficient of variation of the elastic modulus. Typically, this COV ratio is 0.2.

The displacement ratio (the mean displacement of the statistically heterogeneous earth mass divided by the displacement obtained using a homogeneous earth mass having a stiffness equal to the mean) is influenced primarily by the degree of heterogeneity, $V_{\rm e}$. The slope geometry, Poisson's ratio, and the earth pressure coefficient all have a relatively minor influence on the displacement ratio, although these parameters do affect the magnitude of the displacement.

The method used in this study can also be extended to the analysis of earth structures with inelastic nonlinear materials, and may be used where the soil properties vary with depth, or for deposits containing pockets of soft soil.

As a further application of the results shown in this paper, if the failure of a slope is defined in terms of deformation, statistical decision theory can be applied to establish guide lines for determining the number of soil samples required for the design of an excavated slope or for determining the relation between the uncertainty in the mean soil parameter and the failure risk. Also, the number of samples for an acceptance specification of earthwork could be determined. Such examples will be the subject of future papers.

ACKNOWLEDGMENT

The writers wish to express their appreciation to the Alabama Highway Department, the U.S. Department of Transportation, and the Computer Center of Auburn University for assistance in conducting this study. The opinions, findings, and conclusions are those of the authors and not necessarily those of the sponsoring agencies.

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FIELD STUDY OF AN ANCHORED SHEET PILE BULKHEAD

H.Y. FANG* and THOMAS D. DISMUKE+

INTRODUCTION

The section modulus of a structural member is a measure of its ability to resist bending. It is calculated by dividing the area moment of inertia of the member about its axis of bending (the neutral axis) by the distance from that axis to the outermost fiber of the member cross-section. The centroidal axis of an individual arch web sheet piling section is located between the axis of the interlocks and the web (Fig. 1). American design practice is to use this centroidal axis as the neutral axis for evaluating moment resistance. The location of the centroidal axis of a wall composed of several interlocked sheet piling sections is along the line of the interlocks. Some European design practice is to use this axis, or an intermediate position, as the neutral axis to evaluate the moment resistance. As is evident from Fig. 1, the resistance of an individual section is about one half the resistance of the composite group. Consequently, designs based on the European method are more economical than those based on the American method when using arch web piling.

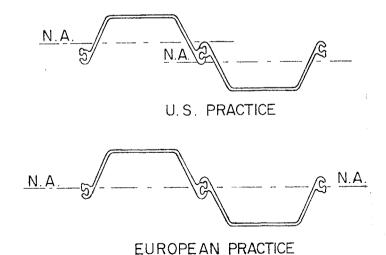


Fig. 1. Location of Neutral Axis (U.S. and European Practice).

^{*} Associate Professor of Civil Engineering and Director, Geotechnical Engineering Division, Fritz Engineering Laboratory, Lehigh University, U.S.A.

⁺ Consulting Engineer, Bethlehem Steel Corporation, Bethlehem, Pennsylvania, U.S.A. *Discussion* on this technical note is open until 1 November 1974.

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A review of the literature concerning sheet pile structures was made by FANG & DISMUKE (1974). The review indicated that there was little factual information relating to the amount of shear transfer across the interlocks located within the extremities of the section. The objective of this study was to evaluate the behavior of interlocked steel sheet piling in an actual field installation. More specifically, the strain distribution across a sheet pile was measured in order to experimentally locate the neutral axis of bending of the wall. This technical note presents a detailed description of the measuring technique and instrumentation used to study the shear transfer in a sheet pile wall.

TEST SITE, INSTRUMENTATION AND MEASURING TECHNIQUES

The test site was located at Martins Creek, Pennsylvania. The soil profile was determined from the results of wash borings and from information supplied by the Pennsylvania Power and Light Company. In general, the test site consisted of a thin layer of sand and silt underlain by a thicker layer of sand, gravel, and boulders.

The foil-type SR-4 strain gage was chosen as the primary means of measuring strain in the piling. As strain gages must be protected during and after pile driving operations, an evaluation of the strain gage system was undertaken in the laboratory. Methods for attaching, waterproofing and protecting the gage were studied. In the laboratory, a gage was mounted on the outer portion of a 2×2 in. angle to simulate the actual mounting of a gage on the sheet piling and driven into soil. An epoxy covering was placed over the gage, but no attempt was made to keep the covering from touching or adhering to the gage. The soil was composed of equal amounts of coarse sand, obtained directly from the test site. The soil mixture was placed in a 2 ft diameter cylinder of 4 ft height. A drain spout was tapped into the bottom and a manometer was attached so that the level of the water table could be controlled and measured.

The laboratory testing of the foil-type strain gage proved that the gage could be successfully protected against abrasion under moderate controlled laboratory conditions, and no trouble would arise as long as the epoxy covering remained unbroken and bonded to the steel. Therefore, care was taken to properly bond the epoxy to the steel in all subsequent gage installations. After it was shown that the gages could be protected under controlled laboratory conditions, it was decided to further test the protective system at a nearby construction project.

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Two gages were attached to a sheet piling in the field and driven 20 ft into a loose, silty sand having a high groundwater table. Both gages were protected by the epoxy covering. One gage was given additional protection by covering it with a steel shoe that was welded to the sheet pile. During and after driving, both gages performed satisfactorily. It was decided to protect all gages with the steel shoe in order to offer additional protection.

The strain gages were installed on the sheet piling in the laboratory prior to delivering the piling to the test site. The piles were cleaned with high speed grinders to obtain a smooth surface for the gages. The ribbon wire was laid flat and clamped before the gage epoxy was applied. After the wires were in place, the gages were attached and clamped while the epoxy was setting. Each gage was checked after installation to ensure adhesion of the gage to the piling. Because of the delicate nature of foil gages, it was necessary to use low temperature solder to install the wires. Terminal tabs were used to allow some play in the wires should they be accidentally pulled. After the wires were installed, electrical readings were taken and the protective epoxy covering was applied.

The strain gages were connected to the arch piles near the interlocks in order to evaluate the shear transfer across the joints. This information would, in turn, lead to the determination of the location of the neutral axis of bending for the sheet pile wall. After the gages were installed on the piling, they were "zeroed" at the laboratory with the piling hanging in a vertical position. The initial readings recorded in the laboratory were checked at random at the site before driving was started on each pile. The comparison between the random checks and the laboratory reading was satisfactory.

The length of the sheet pile wall was 30 ft which was believed to be long enough to minimize undesirable end effects. The piles were 30 ft long and they were driven 25 ft below ground level. Consequently, once the arch piles were in place, approximately 5 ft of pile protruded from the ground surface. A standard driving rig with a low energy double action 9B3 MKT hammer was used for the driving operations. A guide frame was used to insure plumbness of the wall. A transit and a six-foot level were used to aid the positioning of the piles. The wall was anchored by two tie rods which were held back by H-piles driven vertically 20 ft into the ground. The tie rods were attached to the wall by means of a wale welded to the wall at ground level. The wale and tie rods were located at ground level to facilitate instrumentation and test procedures. The field test set-up is illustrated in Fig. 2. It should be noted that two piles met refusal and could not be driven to the required depth. For this reason, they protruded 15 in. above the other piles.

FANG AND DISMUKE

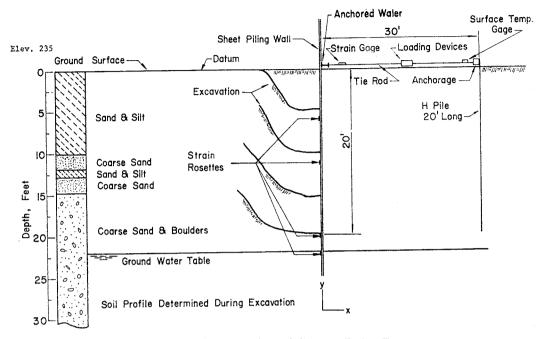


Fig. 2. Cross-Section of Sheet Pile Wall.

The soil was excavated in front of the wall in four stages. Each excavation stage was 5 ft deep. The readings were taken, on all strain gages after the excavation and again one week later, just prior to the next excavation sequence. Excavation time for each stage was one day. This sequence of events was repeated until the excavation reached the 20 ft level. In addition, wall deflections measured by transit were recorded for several wall locations. An initial load of 2000 1bs was applied to each of the tie rods. Detailed field data is summarized and given by Brewer & Fang (1968).

DISCUSSION AND CONCLUSIONS

Figure 4 shows the pertinent data for shear transfer in graphical form. The distribution of vertical strain across the sheet pile interlocks is shown for all gage levels and at all stages of excavation. Although there is considerable scatter in the data and a number of the gages failed completely (see Fig. 3), the graphs suggest that shear stress transfer across the interlocks between piles does occur. This is apparent because the vertical strain distribution across the interlocks may be reasonably approximated by a single continuous straight line at all stages of excavation at which there is sufficient data. If there was only partial or no shear stress transfer across the interlocks, the vertical strain distribution across the piles would be shown by two disconti-

TECHNICAL NOTE ON ANCHORED BULKHEAD

nuous straight lines. Figure 4 shows the vertical strain distribution interlock of joint 1, piles 10 and 11. Other data on piles are given by Brewer & Fang (1968, 1970).

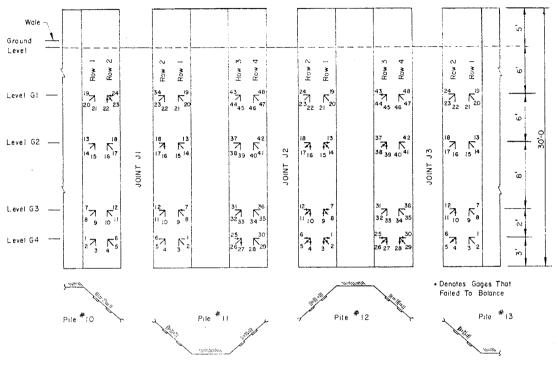


Fig. 3. Location of Strain Gages.

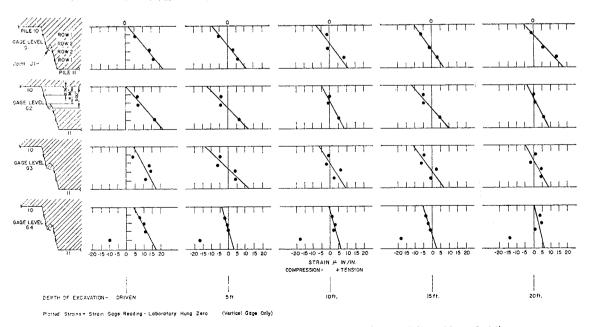


Fig. 4. Vertical Strain Distribution Interlock (Joint 1, Piles 10 and 11).

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It is of interest to note from Fig. 4 that the location of the neutral axis of bending for the sheet pile wall, given by the intersection of the vertical strain curve with the line of zero strain, does not always lie within the pile cross-section. For discussion purposes, consider the behavior of piles Nos. 10 and 11 at gage level G1. It can be seen that just prior to excavation, the neutral axis lies completely outside the piling cross-section toward the fill side of the wall. This could indicate that the piling is in tension due to bending induced during driving, and the compressive bending stresses are carried by the soil behind the piles. Such behavior may be considered to be composite action, with the wall and the soil acting as a unit; however, there is no further substantiation of this.

The analysis of the field study can be summarized as follows, and the conclusions presented are considered applicable within the limitations of the assumptions used:

- 1. Strain gage instrumentation installed on sheet piling prior to driving may be successfully protected against damage during driving.
- 2. Within the range of applied loads and soil conditions encountered in this investigation, the available data suggests that shear transfer takes place across the interlocks of arch web steel sheet piles.

ACKNOWLEDGEMENTS

This investigation was sponsored and financed by the American Iron and Steel Institute. Thanks are extended to Mr. C.E. Brewer who conducted the field test.

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INTERNATIONAL SOCIETY NEWS

The following is an extract of a news letter received from the Secretary General of the International Society, Professor J.K.T.L. Nash.

International Conference Proceedings

5th Conference—Paris, 1965. Now out of print.

Eth Conference—Moscow, 1973. See previous ISSMFE News for details about ordering. The recommended binding is in three volumes as follows

Vol. 1 Parts 1.1, 1.2, 1.3

Vol. II Parts 2.1, 2.2, 3

Vol. III Parts 4.1 etc. (not yet issued)

The binding should be in blue to match that on the paper covers: the lettering in gold should match the London (1957) Conference and should read on the spline (in 3 mm letters):

PROCEEDINGS
EIGHTH
INTERNATIONAL
CONFERENCE
ON SOIL
MECHANICS

MOSCOW 1973 I 1.1, 1.2, 1.3 or II 2.1, 2.2, 3 or III	7 min 3 mm
4.1 etc.	
and on the front of the cover:	
PROCEEDINGS OF THE	4 mm
EIGHTH INTERNATIONAL CONFERENCE	}
ON SOIL MECHANICS AND	5 mm
FOUNDATION ENGINEERING	

MOSCOW	4 mm
1973	

SOUTHEAST ASIAN SOCIETY NEWS

The Fourth Conference on Soil Engineering

The Society's Fourth Conference on Soil Engineering is planned to take place in Kuala Lumpur from 7 to 10 April 1975. The Fourth Conference follows the first three held in Bangkok (1967), Singapore (1970) and Hong Kong (1972): all of these were a great success. The Society Conferences present an opportunity for those interested in soil engineering to spend a few days discussing problems of mutual interest. Participants have found past Conferences to be of great professional value, especially since the papers and discussions have tended to focus on problems of the region.

The Fourth Conference will be sponsored by the Southeast Asian Society of Soil Engineering, the Institution of Engineers, Malaysia, and the Asian Institute of Technology. The language of the Conference will be English. The Organizing Committee has invited Professor T. William Lambe of the Massachusetts Institute of Technology to be the guest lecturer.

There will be no special theme for the Conference. Papers accepted include general topics of soil mechanics and engineering geology dealing with testing and site investigation, foundations, earth dams, slope stability, and roads and runways. The closing date for submission of final manuscripts is 15 July 1974.

Bulletins Nos. 1 and 2, which gives full details about the Conference are available from the Secretary, IV SEACSE, c/o Institution of Engineers, Malaysia, P.O. Box 223, Petalting Jaya, Selangor, Malaysia.

Dr. A.S. Balasubramaniam Becomes Secretary

Dr. A.S. Balasubramaniam was recently appointed Secretary of the Society by the President, Professor Chin Fung Kee. Dr. Balasubramaniam received his B.Sc. in Civil Engineering from the University of Ceylon and, after a few years of practical experience, he carried out research at Cambridge University which led to the award of a Ph.D. degree. He spent one year at the Norwegian Geotechnical Institute in Oslo before returning to Sri Lanka to lecture at the University of Sri Lanka. Dr. Balasubramaniam recently joined the Division of Geotechnical Engineering of the Asian Institute of Technology as Assistant Professor.

CONFERENCE NEWS

The Fourth Southeast Asia Conference on Soil Engineering will be held in Kuala Lumpur, Malaysia from 7 to 10 April 1975. It will be sponsored by the Southeast Asian Society of Soil Engineering, the Institution of Engineers Malaysia, and the Asian Institute of Technology. The language of the Conference will be English. There will be no special theme for the Conference. Papers accepted include general topics of soil mechanics and engineering geology dealing with testing site investigations, foundations, earth dams, slope stability, and roads and runways. Professor T. William Lambe will be the guest lecturer.

Bulletins No. 1 and 2 are available from The Secretary, IV SEACSE, c/o Institution of Engineers, Malaysia, P.O. Box 223, Petaling Jaya, Selangor, Malaysia.

The Sixth European Conference on Soil Mechanics and Foundation Engineering will be held in Vienna in March 1976 on the general theme Deep Foundation and Deep Excavations. The technical sessions will be:

- (1) Deep excavations; stability of temporary and permanent slopes; dewatering problems, slurry walls, walls with batter piles; bracing; freezing techniques.
- (2) Deep foundations; tunnelling.
- (3) Deep foundations in open pits; pile foundations; caisson foundations.

All correspondence should be addressed to the Secretary, VI European Conference on S.M.F.E., A 1040 Wien, Technishe Hochschule, Karlsplatz, 13, Austria.

The 25th International Geological Congress will be held in Sydney, Australia from 16 to 25 August 1976. Those who wish to receive a copy of the first circular when it is available are asked to write to The Secretary-General, 25th International Geological Congress, P.O. Box 1892, Canberra City, ACT 1601, Australia.

The Third International Congress on Rock Mechanies of the International Society for Rock Mechanics will be held in Denver, Colorado, U.S.A. from 1 to 7 September 1974. The object of this Congress is to ascertain on an international scale the advances that have been made in rock mechanics since the Second International Congress, and to indicate directions for future effort. Attention will be centred on the major themes (1) Physical properties of intact

rock and rock massess, (2) Tectonophysics, (3) Surface workings, (4) Underground openings, and (5) Fragmentation systems. At each Plenary Session, the major theme will be reviewed by a Congress General Reporter and a panel will respond to questions submitted prior to the Congress sessions. Selected individual papers will be presented and discussed at the discussion sessions. Brief questions from the floor will be permitted if time allows. The Congress technical sessions, will be presented in English, French and German by simultaneous translation.

Bulletin No. 2 contains details of the Congress, and is now available. This can be obtained from Secretary, Third International Congress on Rock Mechanics, 2101 Constitution Avenue NW, Washington, D.C. 20418, U.S.A.

The Second Australia-New Zealand Conference on Geomechanics will be held in Brisbane, Australia from 21 to 25 July 1975. Topics of sessions will depend on the papers received but it is hoped they will include (1) Measurement Techniques in Soils and Rock Mechanics, (2) Application of Computer Techniques in both Soil and Rock Mechanics, (3) Environmental Aspects of Geomechanics Studies, (4) Design of Earth and Rock Structures, (5) Fracture Mechanics, (6) Foundation Behaviour, (7) Slope Stability, (8) Handling Non-Homogeneous Material, (9) Road and Pavements, (10) Engineering Geology, (11) Groundwater Hydrology.

The closing date for submission of the summaries of the papers was 30 April 1974. Completed manuscripts of the accepted papers must be received by 30 November 1974. All correspondence relating to the Conference should be addressed to The Secretary, 2nd Australia-New Zealand Conference on Geomechanics, 157 Gloucester Street, Sydney, N.S.W. 2000, Australia.

The Second International Congress on Engineering Geology, organized by the Associacao Brasileira de Geologia de Engenharia under the patronage of UNESCO, will be held in Sao Paulo, Brazil from 18 to 24 August 1974. The themes that will be dealt with are (1) Teaching and Training in Engineering Geology, (2) Seismic Phenomena and Engineering Geology, (3) Engineering Geology Related to Urban and Country Plaining, (4) Engineering Properties and Classification of Natural Materials of Construction, (5) Mass Movements, (6) Engineering Geology Relating to Dam Foundations, (7) Engineering Geology and Underground Construction.

The second bulletin is now available from Secretary General, II Congress IAEG, Instituto de Pesquisas Technologicas, Caixa Postal 7141, Sao Paulo—SP, Brazil.

The First Baltic Conference on Soil Mechanics and Foundation Engineering will be held at Gdansk, Poland from 22 to 25 September 1975. The theme of

the conference is "Geotechnics in Hydraulic and Harbour Engineering" and sessions will be devoted to: Theoretical and experimental basis of soil and rock mechanics; shallow foundations; deep foundations; stability of earth structures and structures on bed rock. Papers must be submitted by 30 October 1974.

Further details may be obtained from The Secretary, 1st Baltic Conference on SM and FE, Technical University Institute of Hydro-Engineering, Majakowskiego Street 11, 80-952 Gdansk, P.O.B. 612, Poland.

Australian Tunnelling Conference—The Australian Tunnelling Committee has arranged a conference on Re-Shapping Cities Using Underground Construction. The conference is to be held from 21 to 22 October 1974. Further particulars may be obtained from The Secretary, Australian Tunnelling Conference, Institution of Engineers Australia, 157 Gloucester Street, Sydney, N.S.W. 2000, Australia.

The Second Australian Conference on Coastal Engineering will be held at Gold Coast, Queensland from 27 April to 1 May, 1975. Further particulars may be obtained from The Secretary, 2nd Australian Conference on Coastal Engineering, Institution of Engineers Australia, 157 Gloucester Street, Sydney, N.S.W. 2000, Australia.

The Sixth Regional Conference for Africa on Soil Mechanics and Foundation Engineering will be held in Elangeni Hotel, Durban from 8 to 12 September 1975. The five main sessions are intended to be on: Engineering Geology and Geomorphology; Properties of Soils and Construction Materials; Foundations of Major Structures; Road and Railway Earthworks; Embankment Dams and Dam Foundations. At each of the Main Sessions there will be a General Reporter who, with a small panel, will discuss the topic and refer to papers. Formal contributions of verbal discussions on printed papers will also be arranged at these sessions.

Bulletin No. 1, which gives details about the Conference and the procedure for the submission of papers, is now available from The Organizing Secretary, 6th Regional Conference on Soil Mechanics, c/o N.B.R.I., P.O. Box 395, Pretoria, 001, South Africa.

The First International Conference on Diaphragm Walls and Anchorage Construction sponsored by the Piling Group Committee of the Institution of Civil Engineers, will be held in London from 18 to 20 September 1974. Papers accepted for presentation originated in seven different countries and authors are equally divided between Consulting Engineers and Contractors

personnel with an academic contribution of four appers from Universities. Subjects covered include theory, practice, alternatives, economics, and contractual aspects with records of many case histories and test results. The individual sessions are being arranged so that the major portion of each will be devoted to discussion and to the exchange of information. Application forms and further information can be obtained from the Conference Office, Institution of Civil Engineers, Great George Street, Westminster, London, SW1P 3AA, England.

The Second International Conference on Applications of Statistics and Probability to Soil and Structural Engineering will be held in Aachen, Germany from 15 to 18 September 1975. It is intended to hold four main Conference sessions under the general headings of "Design Philosophy", (a) structures and (b) soils: "Design Parameters", (a) structures and (b) soils.

Bulletin No. 1, which gives details about the Conference and the procedure for the submission of papers, is now available from the Conference Secretariat, Deutsche Gesellschaft fur Erd-und Grundbau e.V., Kronprinzenstrabe 35a, 43 Essen/Federal Republic of Germany.

The Fifth Panamerican Conference on Soil Mechanics and Foundation Engineering will be held at Buenos Aires, Argentina from 27 to 31 October 1975. The conference will consist of six main sessions on the following topics: Stress-Deformation Relationship; Special Soils: Collapsible, Expansive, Preconsolidated by Desiccation; Excavations and Deep Foundations; Tunnels in Soils; Earth and Rockfill Dams; Seismic Design of Earth and Rockfill Dams. A Chairman will lead each Main Session. There will also be a General Reporter, an Adviser and a Panel of specialists.

Bulletin No. 1, which gives details about the Conference and the procedure for the submission of papers, is now available from: Senor, Secretario, Comite Organizador, Mecanica de Suelos e Ingenieria de Fundaciones, Ing. FERNANDO L. TORRES, C.C. 4064, Correo Central, Buenos Aires, Argentina.

The Fifth Asian Regional Conference on Soil Mechanics and Foundation Engineering will be held at the Indian Institute of Science, Bangalore, India from Friday, 19 December to Monday 22 December 1975. The four Main Sessions will be on the topics: (1) Regional Deposits (2) Partially Saturated Soils (3) Foundations and Excavations (4) Structure-Soil Interaction. The Organizing Committee has requested the respective National Societies in the Asian region to submit the summaries of the selected papers not later than 15 November 1974. The deadline for submission of completed papers from the National Societies to the Organizing Committee is 15 March 1975.

Circular 1, giving details about the Conference and the procedure for the preparation of papers can be obtained from Dr. B.V. Ranganatham, Organizing Secretary, 5 ARC, Indian Institute of Science, Bangalore, India.

Istanbul Conference on Soil Mechanics and Foundation Engineering. The Turkish National Society is organising an international conference to take place in Istanbul from 31 March to 2 April, 1975, immediately preceding the ISSMFE Executive Committe meetings on 3 and 4 April. The sessions will be devoted to: (1) Engineering Properties and Behaviour of Soils, (2) Case Studies of Field Behaviour, (3) Analysis and Design in Geotechnics. The conference language will be English and 400 word summaries should be submitted by September 1974, and the full paper by 15 January 1975. Further particulars may be obtained from, Dr. E. Togrol, Zemin Mekanigi Arastirma Kurumu, Teknik Université, Istanbul, Turkey.

NEWS OF PUBLICATIONS

Proceedings of the Regional Conference on Tall Buildings held in Bangkok in January 1974 are available from the Asian Institute of Technology, P.O. Box 2754, Bangkok, Thailand. This volume is edited by prof. S.L. Lee and Dr. P. Karasudhi, and contains 60 selected papers in ten technical sections contributed by specialists from 18 countries. It is hard bound and contains 895 pages. The price, including surface postage, is U.S. \$ 30.

Proceedings of the Second Southeast Asian Conference on Soil Engineering, held in Singapore in June 1970, are available at a price of U.S. \$ 18 (inc. postage) from the Geotechnical Engineering Division, Asian Institute of Technology, P.O. Box 2754, Bangkok, Thailand. Cheques should be made payable to "Asian Institute of Technology".

Proceedings of the Fourth Asian Regional Conference on Soil Mechanics and Foundation Engineering, Bangkok, July 1971 are available in two volumes at U.S. \$ 30 (inc. postage) from the Geotechnical Engineering Division, Asian Institute of Technology, P.O. Box 2754, Bangkok, Thailand.

Geotechnical Abstracts: These abstracts provide a regular worldwide literature information service in the fields of soil mechanics, foundation engineering, rock mechanics and engineering geology. The abstracts are published monthly, at an annual subscription rate of DM 240, by Deutsche Gesellschaft für Erd-und Grundbau, 35a Kronprinzenstrasse 43, Essen, Germany.

Proceedings of the Third Southeast Asian Conference on Soil Engineering, held in Hong Kong in November 1972, are now available at a price of U.S. \$ 25.00 (including postage by registered surface mail). Orders, together with payment, should be sent to the Geotechnical Engineering Division, Asian Institute of Technology, P.O. Box 2754, Bangkok, Thailand.

Proceedings of the Symposium on Role of Plasticity in Soil Mechanics, held in September 1973 at Cambridge University, are now available at a price of £ 4.75 from the Soil Mechanics Group, Cambridge University Engineering Department, Trumpington Street, Cambridge CB2 1PZ, England.

Proceedings of the Symposium on Percolation Through Fissured Rock, held in Stuttgart, Germany in 1972, are now available at a special price of DM 100.00 from Deutsche Gesellschaft für Erd-und Grundbau, 4300 Essen 1, Kronprinzenstrasse 35a, Germany.

ASIAN INFORMATION CENTER FOR GEOTECHNICAL ENGINEERING — AGE

The Asian Information Center for Geotechnical Engineering (abbreviated to AGE) has been established within the library of the Asian Institute of Technology under the joint sponsorship of its Division of Geotechnical Engineering and the Library.

The newly established AGE is an invaluable source of information for all those concerned with investigation, feasibility, design and construction for all types of civil engineering projects. In addition, it is indispensable to those concerned with teaching and research in any aspect of geotechnical engineering. It is aiming to serve as a clearing house in the Asian region for information on Soil Mechanics, Foundation Engineering, Rock Mechanics, Engineering Geology, Earthquake Engineering, and other related fields. In cooperation with national societies, universities, governmental agencies, research organizations, engineering and consulting firms, contractors, etc., both within and outside the region, AGE is collecting information on all phases of geotechnical engineering research and projects, including published and unpublished reports which are of relevance to Asian conditions, AGE is undertaking the responsibility of designing a computer based information storage and retrieval system for the effective handling of such information, and is providing both Current Awareness Service and Selective Dissemination of Information Service through its publication of journal abstracts and subject bibliographies. Dissemination of collected information takes place through photocopying and micro-filming.

To ensure the effective functioning of AGE as an information center for geotechnical engineering in Asia, it is necessary that all organizations and individuals who are engaged in any kind of geotechnical engineering work in Asia consider AGE as a central depository for their information and publications regardless of language. These publications and other information will be abstracted and analyzed, and will be publicized and disseminated through AGE's various channels to the benefit of the region. Among the major data files which have been stated at AGE are:

- (1) Data on all design, construction and research projects in geotechnical engineering of concern to the region.
- (2) Data on organizations engaged in any kind of geotechnical engineering work in the region.
- (3) Data on individuals who are engaged in any kind of geotechnical engineering work in the region.

(4) Data on published papers and technical literature on geotechnical work of concern to the region.

Great benefit can be gained by companies and research organizations treating AGE as a central depository and service agent to provide information when it is needed.

AGE publishes the following:

- (1) Asian Geotechnical Engineering Abstracts: a quarterly publication consisting of abstracts of available publications on geotechnical engineering relevant to Asia.
- (2) Asian Geotechnical Engineering in Progress: a semi-annual publication consisting of information on current design, construction and research projects in geotechnical engineering being undertaken in Asia.
- (3) Asian Geotechnical Engineering Directory: a bi-annual publication consisting of information on various organizations and individuals who are doing geotechnical engineering work in Asia or relevant to Asia.
- (4) AGE Current Awareness Service: published quarterly to inform readers of recent geotechnical engineering publications and contents of geotechnical engineering journals received at AGE.
- (5) AGE Journal Holdings List: published annually to facilitate the request of photocopies.
- (6) AGE Bibliography Series: either recurrent or demand bibliographies published as a result of general interest or demand.

In addition to its publications, AGE provides the following three services:

- (1) Reference Service: for bibiligraphical questions.
- (2) Referral Service: for technical questions.
- (3) Reproduction Service: for photocopying or microfilming of required documents.

AGE is a non-profit making service organization. For the initial three years, it is being financially supported by a generous grant from the International Development Research Centre of Canada and by the Asian Institute of Technology. It will be necessary, however, for AGE to recover a very small portion of its operation costs from fees received on certain services. Much can be gained through membership which is available to Individuals and Institutions at nominal fees.

Anyone who wishes to have details about the Asian Information Center for Geotechnical Engineering should write to Dr. H.W. Lee, Director AGE, P.O. Box 2754, Bangkok, Thailand.

BOOK REVIEWS

Explosives for North American Engineers by C.E. Gregory, Trans. Tech. Publications, Clausthal, Germany, 1973.

This monograph deals with the elementary principles and practices of those particular explosives marketed in North America and used under American and Canadian conditions. The first part of the book describes the nature, composition and methods of firing the various types of explosives. The second part is more practical and deals with the techniques of blasting and their applications in surface excavations, demolition works, underground openings and related fields. Each chapter in the book is followed by a comprehensive list of the most up-to-date references. A useful feature of the book is the inclusion of appendices on safe location specifications for storage of explosives in United States and Canada. The book is concluded with a glossary of technical terms and a combined subject and author index.

This book, though written for North American engineers, will be of considerable interest to all those who are concerned with the use of explosives. Professor Gregory is to be commended for this concise and practical monograph on North American explosives.

A. Kazi

Fundamentals of Earthquake Engineering by N.M. Newmark and E. Rosenblueth, Prentice-Hall Inc., New Jersey, U.S.A., 1973.

This book is divided into three parts. Part I covers dynamics of linear systems, gives a brief introduction to nonlinear systems and a good introduction to hydrodynamics. This part is not meant as a substitute for a classical dynamics course but rather as the first half of a course in structural dynamics. Part II deals with the characteristics of earthquakes, seismicity and the structural response of various systems. This also contains a good description of the probabilistic approach to related problems. Part III deals with the design of earthquake-resistant structures, the recording of ground motions, and test and measurements of structural response. The authors have contributed a large number of research papers in the areas covered by Parts II and III of the book. All the subject matter covered in the book is dealt with fully except for the subjects of soil-foundation interaction and the behavior of soils under dynamic conditions. However, the latter are adequately supplemented with bibliographies for more detailed study.

The book is suitable as a text either for a single course in earthquake engineering with selected portions retained, or for a two-course sequence. It would also be useful for researchers in the field.

Pisidhi Karasudhi

NOTES ON CONTRIBUTIONS TO THIS JOURNAL

Contributions to Geotechnical Engineering are invited from anyone. Items submitted to the Editor will be published under one of the following headings.

Original Papers

Orginal papers should be submitted in accordance with the *Notes for the Guidance of Authors* given inside the back cover of this journal. The Editor undertakes to acknowledge all manuscripts immediately they are received and to arrange for early review of each paper by *two* reviewers. The earliest possible publication date of contributions will be aimed for. Each author will receive 25 free copies of his paper.

Technical Notes

Technical notes will be accepted for publication. These contributions should be presentations of technical information which might be useful to the practicing or research engineer but which are not sufficient in themselves to warrant a full paper. The format to be followed for technical notes is the same as that for papers but only *two* copies need be submitted and no *Synopsis* is required. The author will receive 25 free copies of his technical note.

Reprints

Consideration will be given to reprinting papers which have been published previously but which are unlikely to have come to the attention of Society members. Only papers of a high standard which would be of particular interest to S.E.A.S.S.E. members will be considered.

Discussions

Discussion is invited on any of the papers published in this journal. The closing date for discussion is indicated at the foot of the first page of each paper. Discussions sent to the Editor may be in any form, but figures and references should comply with the general requirements for publications in this journal. Two copies are required.

News Items

As the official organ of the Southeast Asian Society of Soil Engineering, this journal will publish any news item of interest to the Society members. Items to be included in the next issue (June, 1974) should be sent so as to reach the Editor not later than 1 June, 1974.